# Fast Partial Shading Analysis of Large-scale Photovoltaic Arrays via Tearing Method 

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#### Abstract

Partial shading analysis of large-scale photovoltaic (PV) arrays has recently become a theoretically and numerically challenging issue, and it is necessary for PV system designers. The main contributions of this study are the following: 1) A PSIM-based macro-model was employed because it is remarkably fast, has high precision, and has no convergence issues. 2) Three types of equivalent macro-models were developed for the transformation of a small PV sub-array with uniform irradiance to a new macro-model. 3) On the basis of the proposed new macro-model, a tearing method was established, which can divide a large-scale PV array into several small sub-arrays to significantly improve the efficiency improvement of a simulation. 4) Three platforms, namely, PSIM, PSpice, and MATLAB, were applied to evaluate the proposed tearing method. The proposed models and methods were validated, and the value of this research was highlighted using an actual large-scale PV array with 2420 PV modules. Numerical simulation demonstrated that the tearing method can remarkably improve the simulation efficiency by approximately thousands of times, and the method obtained a precision of nearly $6.5 \%$. It can provide a useful tool to design the optimal configuration of a PV array with a given shading pattern as much as possible.


Key words: Partial shading, Photo voltaic (PV) array, Piece-wise linear circuit, Simulation efficiency, Tearing method

## I. Introduction

In the last 10 years, research on photovoltaic (PV) array has focused on the maximum power point (MPP) tracking and module mismatch losses [1] because the partial shading effect of PV arrays has a remarkable influence on the efficiency. A great scale of a PV array corresponds to a great necessary area, and large-area PV arrays have a high possibility to suffer from partial shaded conditions (PSCs) [2]. The average loss in power generation yield within the range of $20 \%-25 \%$ due to PSCs [3]. Therefore, partial shading analysis of large-scale PV arrays has become a theoretically and numerically challenging task.

A model of a PV cell or module is a fundamental to

[^0]understanding the partial shading effects of PV arrays. Over the years, several researchers have studied the characteristics of PV modules. Single- and double-diode PV cell models [4, 5] have been widely used to predict the behaviors of PV cells, modules, short string, and small-scale array. However, these cell models suffer from two drawbacks: (1) their granularity is too fine to handle a long PV module string and a large-scale PV array, and (2) both require a nonlinear implicit equation with exponential terms, leading to computational complexity and convergence issues [6]. A novel PSIM-based macro-model of a PV module was proposed by the researchers of the current study to overcome these drawbacks [3]. Unlike other models, the proposed macro-model is a piecewise linear circuit (instead of nonlinear), which has several considerable advantages, such as fast and high simulation precision, robust execution, and no convergence issues.

Partial shaded analysis started 1995 after Volker Quaschning et al. identified the large loss in a PV array due to PSC [7]. They proposed an appropriate scheme for computing the radiance of the PV cells of a shaded PV array [8]. The
influence of the PV module shading on the performance was reported by the same scholars in 1996 [9]. The performance loss reduced by $70 \%$ when only $2 \%$ of the module area was shaded. The PSC and power losses due to the tracking of local MPPs, instead of the global one, for long, parallel, and multi-string configurations were studied in [10]. Results showed that long series connections of modules and parallel connections of strings of a single inverter to the electrical grid should be curtailed to prevent losses under PSCs. However, these simulation techniques were only suitable to predict the performance of a small-scale PV array, and the simulated maximum PV array was composed of approximately 480 PV modules [2]. Therefore, a high-efficiency approach called tearing approach was required to implement partial shaded analysis of a large-scale PV array for a 500 kW single inverter.

Several types of software and simulation techniques were employed to predict the electrical characteristics of a PV array [10]-[13]. A MATLAB-based model of a PV module was proposed by Walker [13]. On the basis of this model, the characteristics of a PV array with uniform incident irradiance can be obtained. Patel and Agarwal achieved considerable progress in the MATLAB-based model to study the effects of partial shading on PV array characteristics [12]. The model can be used to investigate the effect of temperature and incident irradiance variation and varying shading patterns. Maki and Valkealahti studied power losses in a short string and small-scale array by using 18 PV modules based on the MATLAB platform [10]. However, these simulation techniques use the single-diode PV cell model, thereby suffering from computational complexity and convergence issues. Dash et al. simulated and investigated the PV cells by using a generic model of the PSpice platform [11]. However, these researchers revealed the difficulty faced by the PSpice platform in obtaining results when the number of PV modules was more than 1,000 . Generally, an actual PV system is composed of a PV array and a DC/AC inverter, which are types of power electronic circuits. Therefore, the PV module model should conveniently be interfaced with the models of the actual PV system to estimate the performance of different topologies and control strategies. Unlike PSpice, MATLAB, Saber, and Simplorer, PSIM is an efficient software for simulating a power electronic system, whose devices are an ideal model with a high running speed [14], [15]. Therefore, the PSIM-based macro-model was selected as the main model in the present study.

In an actual PV system, the normal power generation of a PV inverter is 500 kW . In this case, a large-scale array, typically composed of six to seven junction boxes, is required to act as the input DC source of the PV inverter. Each junction box connects 16 standard PV strings in parallel. A standard PV string is composed of 22 PV modules connected in series. Therefore, more than 2,200 PV modules are involved in a large-scale array. Hence, a tearing method is proposed in

Section V of this paper to analyze the performance of a large-scale PV array under PSCs.

The configuration of a PV array is another major issue that influences its performance under PSCs. Currently, several researchers are focused on reconfiguration of PV modules in a PV array. Various PV array configurations under PSCs, such as series-parallel, honey comb, bridge link, and total cross-tie, were presented in [16]. The tearing method proposed in this paper could be expected to analyze the performance of PV arrays in these configurations.

A fast partial shading analysis of large-scale PV arrays via the tearing method was presented in this paper. Three models for PV modules, i.e., a physical model in the PSIM Library, single diode, and PSIM-based macro-model, were introduced in Section II. These models were compared in Section III by using the Yingli YL-235-29b module, obtaining several valuable results. Three types of equivalent macro-models were presented in Section IV to simplify the simulation of series, parallel, and series-parallel PV modules with uniform irradiance. On the basis of the proposed equivalent macromodels, a tearing method for large-scale PV arrays was introduced in Section V. Finally, three different platforms (i.e., PSIM, PSpice, MATLAB) were employed in Section VI to evaluate the proposed tearing method.

Simulated examples of two large-scale PV arrays were established to validate the proposed models and methods. A PV array with 440 modules was employed as the first example, and results demonstrate that the tearing method can remarkably improve the simulation efficiency of approximately 4,860 times and maintain a simulated precision of approximately $5 \%$. An actual large-scale PV array with 2,420 modules was another example, and the simulation indicated that computing time was approximately 1.5 s and simulation error was approximately $6.5 \%$. However, the computer cannot obtain the results due to insufficient memory if all the models introduced in this paper, such as model in PSIM Library, PSIM-based macro-model, and compact model, were directly used to simulate the actual PV array. Hence, the tearing method is intended to serve as a useful tool to design the most optimal configuration of the PV array if a shading pattern is known.

## II. Review of Models for the PV Module

## A. Models in the PSIM Library

PSIM provided two types of PV models, namely, functional and physical, in its library. The functional model was simple and easy to use, but it was difficult to apply in simulating a PV string. The physical model considered the effects of the incident irradiance and temperature, which were important parameters for predicting the characteristics of a PV module. Furthermore, the physical model allowed users to input detailed parameters from the PV cell's datasheet. PSIM can


Fig. 1. Physical model of PV cell and module with the symbols in the PSIM Library: (a) PV cell model, (b) The number, $N_{s}$, of PV cells is connected in series to form a PV module, (c) PV module symbol in the PSIM Library.
also facilitate the extraction of model parameters by using the PV module tool from a manufacturers' datasheet. Hence, the process of modeling and analyzing an actual power system was simplified.

Fig. 1(a) presents the physical model of a PV cell. A PV module was composed of a number $\left(\mathrm{N}_{\mathrm{s}}\right)$ of PV cells connected in series, as shown in Fig. 1(b). Fig. 1(c) illustrates the PV module symbol in the PSIM Library, where S is the incident irradiance and $T_{a}$ is the ambient temperature. In Section III of this work, the physical model was employed to facilitate the partial shading analysis of a PV array. The simulation showed that the physical model in the PSIM Library can predict the characteristics of a large-scale PV array under uniform irradiance, but the model was unsuitable for non-uniform irradiance conditions.

## B. Compact Model

Although the physical model is simplified and easy to use, a compact or effective model of a PV module is required for two main reasons [18]. First, as the number of PV cells in a series increases, so does the number of nonlinear equations of describing the physical model. An excessive number of equations may cause an unreasonably long simulation runtime and decrease the simulation precision. Second, given that the scaling rules of current and voltage are known and become generally constant, developing a compact model for a PV module is simple and useful. In accordance with a singlediode model of a PV cell [Fig. 1(a)] and considering its accuracy and complexity [19], a compact PV model of a PV module, instead of a PV cell, has been extrapolated [13] as

$$
\begin{equation*}
I_{M}=I_{s c M}\left[1-\exp \left(\frac{V_{M}+I_{M} R_{s M}-V_{o c M}}{N_{s} V_{T} A}\right)\right] \tag{1}
\end{equation*}
$$

where M denotes the module, $I_{M}$ is the module current, $V_{M}$ is the module voltage, $I_{s c M}$ is the short-circuit current, $V_{o c M}$ is the open-circuit voltage, $R_{S M}$ is the series resistance, $V_{T}$ is the junction thermal voltage of the PV cell, $A$ is the diode ideality constant, and $N_{s}$ is the number of cells connected in series in the module. However, the compact model was based on the simplified assumption that the shunt resistance, $R_{s h M}$, of a PV


Fig. 2. Alternative compact model of a PV module (i.e., a singlediode model).


Fig. 3. $V-I$ characteristic curve of an actual PV and three remark points, namely, short-circuit current $I_{s c M}$, MPP voltage $V_{m p M}$ and MPP current $I_{m p M}$, and open-circuit voltage $V_{o c M}$.
cell is large and that its effects can be neglected, which will adversely affect the simulation precision.

An alternative compact model of a PV module, referred to as a single-diode model in this paper, was presented by Villalva et al., as shown in Fig. 2 [19]. It can be used to simulate the performance of a PV array. The parameter equations in the single-diode model can be obtained by scaling the parameters used in the one-diode model of a single PV cell. In the model, $R_{s M}$ is the equivalent series resistance of the PV module and $R_{\text {shM }}$ is the equivalent parallel resistance. The effect of both resistances assures that the maximum power of the model matches the MPP of the actual PV module. This model was obtained from the voltage-current ( $V-I$ ) characteristic curve in Fig. 3, where three remarkable points are highlighted: short-circuit current $I_{s c M}$, voltage $V_{m p M}$ and current $I_{m p M}$ at the MPP, and open-circuit voltage $V_{o c M}$. The main aim of this model is to directly obtain its parameters by using the three remark points and several typical test data in the manufacturer's datasheet.

In Section III, an attempt is made to use the single-diode model to facilitate a partial shading analysis of a large-scale PV array. Simulation results verify that the single-diode model of a PV module can predict the performance of a PV array under uniform and non-uniform irradiance conditions. However, as the scale of the PV array increases, the simulation runtime increases and the computational errors becomes too large to be accepted.

## C. PSIM-based Macro-Model

A PSIM-based macro-model of a PV module [17] (Fig. 4) was developed according to its manufacturer datasheet information, which contains several key parameter values (i.e., $V_{o c M}, I_{s c M}, V_{m p M}$, and $I_{m p M}$ ) and the maximum power generation (i.e., $P_{\max M}$ ), as well as several temperature coefficients under standard test conditions (STCs). This macro-model uses only two ideal diodes ( $D$ and $D_{b p}$ ), two resistors ( $R_{s M}$ and $R_{s h M}$ ), two independent voltage sources ( $E_{l}$ and $E_{2}$ ), and an independent current source ( $I_{\text {scM. }}$ ).

The macro-model focuses on the three remarkable points, namely, short circuit ( $0, I_{s c M}$ ), MPP ( $V_{m p}, I_{m p}$ ), and open circuit ( $V_{o c M}, 0$ ), and its circuit parameters are expressed as

$$
\begin{gather*}
E_{1}=V_{o c M}  \tag{2}\\
E_{2}=N_{p b} V_{o n} .  \tag{3}\\
I_{p v}=I_{s c M} .  \tag{4}\\
R_{s M}=\frac{V_{o c M}-V_{m p}}{I_{m p}} . \tag{5}
\end{gather*}
$$

Fig. 5 shows a typical $V-I$ curve of a commercial PV module, Yingli YL-235-29b, provided by Yinglin Solar datasheets [21], where the curves (1) and (2) are the experimental $V-I$ and $V-P$ curves under STCs, respectively. If the $V-I$ curve is split into three regions, that is, the voltage source region ( $V S R$ ), current source region ( $C S R$ ), and the bypass diode conduction region ( $B D C R$ ), then the macromodel can be approximately expressed in three asymptotic lines. The lines are denoted by the broad-brush dashed-line curve (3), replacing the experimental $V-I$ characteristic curve (1). The main advantage of this model is its ability to facilitate the extraction of the parameters of the PV module model by using several key parameter values in the manufacturer's datasheet.

Given that the short-circuit current of the PV module is directly proportional to the incident irradiance value of $G$ $\left(\mathrm{W} / \mathrm{m}^{2}\right)$ and has a small positive temperature coefficient of $\alpha_{\text {isc }}\left(0.05 \%{ }^{\circ} \mathrm{C}\right)$, the short-circuit current for various irradiance and temperature values can be calculated as [3]

$$
\begin{align*}
& I_{s c M}=I_{s c M r}\left[\frac{G}{1000}+\alpha_{i s c}\left(0.035 G+T_{a}-T_{r}\right)\right]  \tag{6}\\
& T_{r}=25^{\circ} \mathrm{C}, \alpha_{i s c}=0.05 \% /^{\circ} \mathrm{C}
\end{align*}
$$

The open-circuit voltage has a negative temperature coefficient $\beta_{\text {voc }}$ and a minimal dependence on irradiance. Hence, Equation [3] was formulated

$$
\begin{align*}
V_{o c M} \approx & V_{o c M r}\left[1+\beta_{v o c}\left(0.035 G+T_{a}-T_{r}\right)\right] \\
& +\frac{V_{T} T_{a}}{T_{r}}\left(\frac{I_{s c M}}{I_{s c M r}}-1\right), V_{T}=0.026 \mathrm{~V} \tag{7}
\end{align*} .
$$



Fig. 4. PSIM-based macro-model of the PV module.


Fig. 5. $V-I$ curve of a commercial PV module: Experimental $V-I$ curve (1) and $V-P$ curve (2) under STCs; the $V-I$ curve of the PSIM-based macro-model (3).

Similarly, the voltage and current at MPP for different irradiance and temperature values are expressed [3] as

$$
\begin{gather*}
I_{m p} \approx I_{m p r} \frac{G}{1000} .  \tag{8}\\
V_{m p} \approx V_{m p r}\left[1+\beta_{v m p}\left(0.035 G+T_{a}-T_{r}\right)\right] \\
+\frac{V_{T} T_{a}}{T_{r}}\left(\frac{I_{s c M}}{I_{s c M r}}-1\right) \tag{9}
\end{gather*} .
$$

Formulas (2)-(9) are used to compute the key parameters of the macro-model for an arbitrary value of the incident irradiance and ambient temperature.

Unlike the two other models, the macro-model is a piecewise linear circuit that has several considerable advantages, such as high simulation accuracy and short computing time, but it also has limitations. For example, as the scale of the PV array increases to 440 modules, the simulation runtime become longer than the acceptable time, but the computational error remains less than $5 \%$.

## III. COMPREHENSIVE COMPARISON WITH THE THREE TYPICAL MODELS

In this section, the Yingli YL-235-29b module was employed to evaluate and investigate the three typical models, namely, the physical model in the PSIM Library, the single-diode model, and the PSIM-based macro-model, introduced in


Fig. 6. Simplest PV string composed of two modules connected in series with different incident irradiances: (a) PV string configuration, (b) Simulation $V-P$ characteristic curves by using a single-diode model and PSIM-based macro-model, denoted by (1) and (2).

TABLE I
Simulation Errors at MPP for Three Typical PV Module Models

|  | PHYSICAL MODEL | Single-diode model | Macro-model |
| :--- | :---: | :---: | :---: |
| $\mathrm{V}_{\mathrm{mpM}}$ | $-2.7 \%$ | $-0.2 \%$ | $-0.8 \%$ |
| $\mathrm{I}_{\mathrm{mpM}}$ | $+0.2 \%$ | $-2.2 \%$ | $+3.2 \%$ |
| $\mathrm{P}_{\mathrm{mp}}$ | $-2.5 \%$ | $-2.4 \%$ | $+2.5 \%$ |

Section II. The Yingli datasheet parameters were specified and the $V-I$ curve families with different incident irradiances under STCs were employed as the experimental data, which were used to compute the simulation errors in this paper. A personal computer was used to run the PSIM programs with a step of $10^{-7} \mathrm{~s}$, test the runtime, and estimate the accuracy of the different models. PSIM is designed for power electric circuits in which the devices mainly adopt the ideal model, with a fast design speed and great flexibility in modeling. The simulation of the PV array using PSIM software has high accuracy, which can accurately reflect its physical characteristics [10].

## A. Uniform Irradiance Condition

With the use of the three typical PV module models, the $V-$ $I$ and $V-P$ characteristic curves of the single PV module were simulated under STCs. The simulation errors at MPP are listed in Table I. The maximum error was approximately $\pm 2.5 \%$. This result confirms that the accuracy of the three typical PV module models can meet the requirements of the routine engineering analysis and design. Moreover, all simulation times of the three typical models did not exceed 1 s ; hence, discussion was not necessary. Note that the simulation error and time are two important specifications that determine whether the model can be used to simulate a large-scale array.

## B. Non-uniform Irradiance Condition

Fig. 6(a) shows the simplest PV string composed of two modules connected in series under non-uniform irradiance conditions. The PV string was subjected to various incident irradiances, that is, 1,000 and $600 \mathrm{~W} / \mathrm{m}^{2}$. If the physical model in the PSIM Library is employed, then the runtime will


Fig. 7. Long PV string composed of 22 modules under a nonuniform irradiance condition.
exceed 15 min and show an error, and no simulation result can be obtained. Thus, the physical model is unsuitable for the non-uniform simulation of a PV array because the bypass diode is not present in the physical model. Fig. 6(b) illustrates the simulation of the single-diode model and macro-model, denoted by (1) and (2), respectively. The double-peak points of MPP1 and MPP2 appeared in the $V-P$ curves because the modules in this PV string had different incident irradiances. The maximum error at MPP was less than $\pm 5 \%$, and all the runtimes did not exceed 1 s . These results further verified that the accuracy of the two typical models was sufficiently good to predict the behavior of a PV string in non-uniform irradiance conditions.

## C. Long PV String in Non-uniform Irradiance Condition

Fig. 7 shows a long PV string with 22 PV modules connected in series under non-uniform irradiance conditions. The modules involved in this string are divided into two groups. The first group is the full-sun PV modules with the irradiance value $G_{l}\left(1,000 \mathrm{~W} / \mathrm{m}^{2}\right)$ and number $N_{l}(11)$; the second group is the fully shaded PV modules with $G_{2}(600$ $\mathrm{W} / \mathrm{m}^{2}$ ) and $N_{2}(11)$. The long string is employed to estimate the behaviors of a standard PV module string operated at two incident irradiance values. The single-diode model and macro-model of a PV module are used during the simulation process because the physical model is unsuitable for non-

TABLE II
Simulation Errors of a Long PV String with 22 PV Modules in Non-uniform Irradiance Conditions

|  | Single-diode model | Macro-model |
| :---: | :---: | :---: |
| $V_{m p M 1}$ | $-12.69 \%$ | $-1.1 \%$ |
| $I_{m p M 1}$ | $-2.4 \%$ | $+3.2 \%$ |
| $P_{m p M 1}$ | $-14.7 \%$ | $+2.1 \%$ |
| $V_{m p M 2}$ | $-11.4 \%$ | $-4.4 \%$ |
| $I_{m p M 2}$ | $+1.3 \%$ | $+0.05 \%$ |
| $P_{m p M 2}$ | $-10.1 \%$ | $-4.42 \%$ |



Fig. 8. Small PV array composed of 20 PV strings in non-uniform irradiance conditions.
uniform irradiance of a PV array. The simulation errors of the long PV string are listed in Table II. As shown in the table, the maximum error of the macro-model is $-4.42 \%$, and the runtime is approximately 2 s . Both models obtained excellent results that are within the acceptable range. However, the maximum error is enhanced up to $-14.7 \%$ and the runtime increases up to 50 s if the single-diode model is used. Therefore, the single-diode model is unsuitable for simulating a long PV string and a large-scale array under non-uniform irradiance conditions given that either time consumption or precision is not within the acceptable range.

## D. Small-scale Array in Non-uniform Irradiance Condition

Fig. 8 shows the configuration of a small PV array consisting of 22 PV strings (Fig. 7) connected in parallel in non-uniform irradiance conditions; hence, no 440 PV modules were involved in the array. The configuration was employed to predict the performance of small-scale PV array under non-uniform irradiance conditions. The macro-model of the PV module was applied in the simulation because the physical and single-diode models were unsuitable for simulating a PV array in non-uniform irradiance conditions. The simulation showed the time consumption of 81 min and more than 1 h , and the maximum error at MPP was $-4.6 \%$.

The macro-model has several advantages over the two other models, such as high simulation precision and short runtime. However, it still exhibited several limitations; for


Fig. 9. Piecewise linear equivalent circuit for the PSIM-based macro-model of the PV module: (a) BDC region, (b) CS region, (c) VS source.
example, it was too consuming to be acceptable for a largescale PV array.

## IV. EQUIVALENT MACRO-MODEL FOR SERIES AND Parallel PV Modules

In a large-scale PV array, many PV modules were connected in series to form a PV string to improve the DC bus voltage, whereas several strings were connected in parallel to form a PV array to increase the power generation. In this section, three types of equivalent macro-models were developed to simplify the simulation of series, parallel, and series-parallel PV modules with uniform irradiance.

## A. Series Macro-model

Fig. 9 shows a piecewise linear equivalent circuit for a PSIM-based macro-model of the PV module (Fig. 4). Figs. $9(\mathrm{a}), 9(\mathrm{~b})$, and $9(\mathrm{c})$ show the corresponding linear equivalent circuits of the PV module if the operating point (OP) remained at the $V S R, C S R$, or $B D C R$, respectively. The three PV modules with uniform irradiance were connected in series, as shown in Fig. 10(a). The current passed through these PV modules with the same value. As a result, their OPs remained in an identical region.

When the OP remains at the $V S R$, the original equivalent circuits [Fig. 9(c)] will be connected in series to achieve an equivalent circuit. With the application of superposition theory for a linear circuit, the parameters of the equivalent circuit can be expressed as

$$
\begin{gather*}
R_{s S}=N_{s} R_{s M}  \tag{10}\\
E_{1 S}=N_{s} V_{o c M} \tag{11}
\end{gather*}
$$

where S denotes the string and $\mathrm{N}_{\mathrm{s}}$ is the number of PV modules connected in series.

When OP remains at the $C S R$, the original equivalent circuits [Fig. 9(b)] will be connected in series to form an equivalent circuit. Note that the series resistance $R_{s M}$ is much smaller than $R_{\text {shM }}$ and its effects can be neglected. Hence, the parameters of the equivalent circuit are expressed as


Fig. 10. (a) Three PV modules with uniform irradiance connected in series, (b) Series equivalent macro-model.

$$
\begin{align*}
R_{s h S} & =N_{s} R_{s h M} .  \tag{12}\\
I_{s c S} & =I_{s c M} . \tag{13}
\end{align*}
$$

Similarly, the parameter of the new equivalent circuit in the BDCR can be obtained by

$$
\begin{equation*}
E_{2 S}=-N_{S} E_{1} \tag{14}
\end{equation*}
$$

Consequently, a PV string composed of $N_{s} \mathrm{PV}$ modules with uniform irradiance connected in series can be simulated as a series equivalent circuit, as shown Fig. 10(b). Its parameter can be calculated using Formulas (10)-(14).

## B. Parallel Macro-model

The three PV modules with uniform irradiance connected in parallel are shown in Fig. 11(a). Fig. 11(b) presents the parallel equivalent macro-model according to the conclusion elaborated in Section IV- $A$ because the voltages across these PV modules had equal values and the $V-I$ characteristics had a monotonic function. If $N_{p} \mathrm{PV}$ modules with uniform irradiance were connected in parallel to form a parallel equivalent macro-model that can be used to describe the $V-I$ characteristics, then the parameter values can be determined by

$$
\begin{gather*}
R_{s P}=\frac{R_{s M}}{N_{p}}  \tag{15}\\
E_{1 P}=V_{o c M}  \tag{16}\\
R_{s h P}=\frac{R_{s h M}}{N_{p}}  \tag{17}\\
I_{s c P}=N_{p} I_{s c M}  \tag{18}\\
E_{2 S}=-E_{1} \tag{19}
\end{gather*}
$$



Fig. 11. (a) Three PV modules with uniform irradiance in parallel, (b) Parallel equivalent macro-model.


Fig. 12. Several PV modules with uniform irradiance connected in series-parallel and their equivalent macro-model: (a) PV array configuration of several PV modules with uniform irradiance connected in series-parallel, (b) Series equivalent macro-model, (c) Series-parallel equivalent macro-model.
where $P$ denotes the parallel module, and $N_{p}$ is the number of PV modules connected in parallel.

## C. Series-parallel Macro-model

Fig. 12 shows a series-parallel equivalent macro-model for a PV array with uniform irradiance. Two strings were connected in parallel, and each string was composed of three PV modules connected in series, as shown in Fig. 12(a). With the use of the proposed series equivalent macro-model, the circuit shown in Fig. 12(a) can be simplified as a circuit of Fig. 12(b). Similarly, the circuit shown in Fig. 12(b) can be simplified as a circuit of Fig. 12(c) according to the proposed parallel equivalent macro-model. Therefore, a large-scale PV array with uniform irradiance can be modeled as a seriesparallel equivalent macro-model, which effectively reduces computing time.

A large-scale PV array with uniform irradiance is assumed to contain $N_{p} \mathrm{PV}$ strings, and each PV string is composed of $N_{s} \mathrm{PV}$ modules. Its macro-model is illustrated in Fig. 12(c). The parameters can be obtained using the following formulas:

$$
\begin{align*}
& R_{s A}=\frac{N_{s} R_{s M}}{N_{p}},  \tag{20}\\
& E_{1 A}=N_{s} V_{o c M},  \tag{21}\\
& R_{s h A}=\frac{N_{s} R_{s h M}}{N_{p}},  \tag{22}\\
& I_{s c A}=N_{p} I_{s c M},  \tag{23}\\
& E_{2 A}=-N_{s} E_{1}, \tag{24}
\end{align*}
$$

where the subscript A is the array, $N_{s}$ is the number of PV modules in the string, and $N_{p}$ is the number of PV strings in a PV array.

## V. TEARING METHOD FOR LARGE-SCALE PV ARRAY

Diakoptics (Greek dia [through] + kopto [cut, tear]) or the "method of tearing" [22] involves partitioning large-scale electric networks into computationally small sub-circuits [23], [24]. In the current study, a large-scale PV array was split into several PV sub-systems, such as sub-string or sub-array. Hence, the involved PV modules received uniform incident irradiance. Each PV sub-system was established as a new equivalent macro-model, referred to as the macro-model of the PV sub-system. Moreover, all new macro-models will constitute the model of the entire PV array.

## A. PV Array with Two Values of Irradiance

A PV array with two values of irradiance is shown in Fig. 13(a), sharing the same configuration with Fig. 8. A total of 20 PV strings are involved, as shown in Fig. 7. Each string is composed of two groups. The first group is the full-sun PV modules with the irradiance value $G_{l}\left(1,000 \mathrm{~W} / \mathrm{m}^{2}\right)$ and number of $N_{s l}(11)$; the second group is the fully shaded PV modules with $G_{2}\left(600 \mathrm{~W} / \mathrm{m}^{2}\right)$ and $N_{s 2}(11)$.


Fig. 13 Original PV array with two irradiance values and tearing process, as well as its simplified equivalent macro-model: (a) Original PV array with two irradiance values, (b) Sub-string composed of PV modules with identical irradiance connected in series, denoted by $\operatorname{sub}_{\mathrm{i}}{ }^{\mathrm{j}}$, where the subscript $i$ represents $i$ th sub-string, and the superscript $j$ is the number of strings in the array, (c) All identical potential points are connected using a wire, (d) A simplified equivalent macro-model of a PV array: Sub1 and $\boldsymbol{S u b} 2$ represent the $N_{p}(20) \operatorname{sub}_{1}{ }^{\mathrm{j}}$ and $\operatorname{sub}_{2}{ }^{\mathrm{j}}$, respectively, connected in parallel.

With the application of the proposed series equivalent macro-model in Section IV- $A$, a sub-string with identical irradiance can be simplified as a macro-model, denoted by $s u b_{\mathrm{i}}{ }^{\mathrm{j}}$, where subscript $i$ represents $i$ th sub-string and

TABLE III
Simulation Error of the Simplified Macro-model

|  | Macro-model |
| :---: | :---: |
| $V_{m p M 1}$ | $-3 \%$ |
| $I_{m p M 1}$ | $+0.2 \%$ |
| $P_{m p M 1}$ | $+2.8 \%$ |
| $V_{m p M 2}$ | $-4.67 \%$ |
| $I_{m p M 2}$ | $-0.06 \%$ |
| $P_{m p M 2}$ | $-4.55 \%$ |

TABLE IV
Simulation Error Between the Original PV Array and Simplified Macro-model

|  | Original PV array and <br> simplified macro-model |
| :---: | :---: |
| $V_{m p M 1}$ | $-0.7 \%$ |
| $I_{m p M 1}$ | $+0.2 \%$ |
| $P_{m p M 1}$ | $-0.5 \%$ |
| $V_{m p M 2}$ | $-0.09 \%$ |
| $I_{m p M 2}$ | $+0.6 \%$ |
| $P_{m p M 2}$ | $-0.5 \%$ |

superscript $j$ is the number of strings in the array. Hence, the new configuration of the PV array can be obtained and shown in Fig. 13(b).

As shown in Fig. 13(b), a current source is connected in the media points between the two adjacent sub-strings. The simulation results demonstrate that the currents $I_{1} \ldots I_{19}$ have identical values with zero, thereby indicating that all media points have equal potential because the PV array is a symmetrical circuit. Hence, its behaviors cannot be changed if a wire is used to connect all media points. Hence, the PV array shown in Fig. 13(b) can be simplified as a circuit of Fig. 13(c).

With the application of the proposed parallel equivalent macro-model in Section IV-B, a simplified equivalent macromodel can be established and shown in Fig. 13(d). The new equivalent circuit is composed of two sub-macro circuits, referred to as $\boldsymbol{S u b 1}$ and $\boldsymbol{S u b 2}$. Sub1/Sub2 represents the $N_{p}$ (20) strings connected in parallel, and each string includes the $N_{s l} / N_{s 2}$ (11) PV modules with $1,000 / 600 \mathrm{~W} / \mathrm{m}^{2}$ irradiance value connected in series. The simplified model parameters can be obtained by using Formulas (20)-(24), where $N_{S}=11$, $N_{p}=20$.

As shown in Fig. 13, with the use of the proposed equivalent circuits, the original PV array with 440 PV modules [Fig. 13(a)] can be simplified as two macro-models in series [Fig. 13(d)]. When the PSIM-based macro-model is used to directly simulate the original PV array, the time consumption is approximately 81 min . However, the computing time is less than 1 s if the simplified macro-model is employed. Therefore, the proposed tearing method can improve the simulation efficiency by approximately 4,860 times.

Table III shows the simulation error of the simplified


Fig. 14. Simulation using the original PV array and its simplified equivalent macro-model.

(a)

(b)

Fig. 15. Diagram of an actual large-scale PV array with three irradiance values and its simplified equivalent macro-model: (a) Diagram of an actual large-scale PV array with three irradiance values, (b) Simplified equivalent macro-model of an actual large-scale PV array, where $\boldsymbol{S u b}_{\boldsymbol{A}}, \boldsymbol{S u b}_{\boldsymbol{B}}$, and $\boldsymbol{S u b _ { \boldsymbol { c } }}$ represent Sub-array 1, Sub-array 2, and Sub-array 3.
macro-model. As shown in the table, the maximum error appeared at MPP2, which was less than $5 \%$. Thus, the proposed tearing method has acceptable simulation precision. Fig. 14 shows the simulated $V-P$ characteristic curves obtained by using the original PV array and its simplified equivalent macro-model. Curves (1) and (2) represent the original PV array and its simplified equivalent macro-model, respectively. The two curves nearly coincide with each other. The simulation error between the original PV array and simplified macro-model is presented in Table IV, in which the maximum error is only $-0.7 \%$.

## B. Actual Large-scale PV Array

In the actual PV system mentioned in Section I, the normal power generation yield of a PV inverter is 500 kW . Fig. 15 illustrates a diagram of the actual large-scale PV array with three irradiance values, in which $2,420 \mathrm{PV}$ modules and 110 strings are involved. The entire array can be divided into


Fig. 16. Simulation by using the simplified equivalent macromodel.
three sub-arrays: Sub-array 1, Sub-array 2, and Sub-array 3.
Sub-array 1 involves 30 PV strings connected in parallel. Each string consists of two sub-strings connected in series, denoted by $\operatorname{Sub}^{1}{ }_{1}$ and $\operatorname{Sub}{ }_{2}{ }_{2}$. These sub-strings have $N_{s l} / N_{s 2}$ (11) PV modules with $G_{l} / G_{2}\left(1,000 / 600 \mathrm{~W} / \mathrm{m}^{2}\right)$ irradiance values, respectively. With the use of the simplified equivalent model in Section V- $A$ (Fig. 13), Sub-array 1 can be transformed into two macro-models connected in series, denoted by $\mathrm{Sub}_{\mathrm{A}}$, as shown in Fig. 15(b). Similarly, Sub-array 2 can be transformed into $\mathrm{Sub}_{\mathrm{B}}$, as shown in Fig. 15(b). Note that $G_{3}$ is equal to $200 \mathrm{~W} / \mathrm{m}^{2}$.

Sub-array 3 includes 50 PV strings connected in parallel, and each string consists of 22 PV modules with $1,000 \mathrm{~W} / \mathrm{m}^{2}$ irradiance value. Therefore, it is a sub-array with uniform irradiance. $\boldsymbol{S u b}_{\boldsymbol{c}}$ represents the Sub-array 3 in Fig. 15(a) obtained by applying the series-parallel macro-model in Section IV-C.

Fig. 16 illustrates the $V-P$ characteristic curve of the actual large-scale PV array [Fig. 15(a)] when the simplified macromodel [Fig. 15(b)] is employed. The computing time is approximately 1.5 s , and the simulation error is approximately $6.5 \%$. Hence, the proposed tearing method can remarkably improve the simulation efficiency, and its error is acceptable. However, the computer cannot obtain the results due to limited memory if the PSIM-based macro-model is directly used to simulate the original PV array.

## C. Computing Time

The diagram of an actual large-scale PV array [Fig. 15(a)] is employed to compare the computing times versus the number of PV modules. In the computing process, the configuration of the PV array is maintained, but the number of module changes as the number of strings also changes. Hence, the numbers of selected PV modules were 200, 400, 800 , 1,210, and 2,420. Fig. 17 shows a comparison between the computing time and the number of PV modules in an array. Fig. 17 shows the two curves. Curve (1) represents the time consumption of the PSIM-based macro-model, and curve (2) is the computing time of the proposed tearing method. Table V shows the computing time of different numbers of PV modules. As shown in the table, the proposed tearing method remarkably improves the simulation efficiency.


Fig. 17. Computing time versus number of PV modules in an array: PSIM-based macro-model(1), Tearing method(2)).

TABLE V
Computing Time with Different Numbers of Module

| Number <br> of modules | $A_{N}$ <br> $(N=1-5)$ | $B_{N}$ <br> $(N=1-4)$ |
| :---: | :---: | :---: |
| 200 | 1 s | 4 min |
| 400 | 1.1 s | 76 min |
| 800 | 1.2 s | 2 hours |
| 1210 | 1.3 s | 3 hours |
| 2420 | 1.5 s | N/A |

TABLE VI
Configuration Cases

|  | Sub-array 1 | Sub-array 2 | Sub-array 3 |
| :---: | :---: | :---: | :---: |
| CASE 1 | 5 | 10 | 25 |
| CASE 2 | 15 | 15 | 25 |
| CASE 3 | 30 | 30 | 30 |

## VI. TEARING METHOD ApPlication of DIFFERENT PLATFORMS

In this section, PSIM, PSpice, MATLAB were employed to evaluate the proposed tearing method. The configuration of the testing array (Fig. 15) is still used as the testing array. Three cases are considered, and the configurations are shown in Table VI. For example, in Case 1, a total of 880 PV modules are present in the array. Sub-array 1 has five PV strings, Sub-array 2 has 10 PV strings, and Sub-array 3 has 25 PV strings.

The simulation results are shown in Table VII. As shown in the table, the use of the tearing method effectively increased the scale of the simulated array while improving the simulation efficiency and maintaining high precision. On the Pspice platform, although the largest number of simulated array modules increases from 880 to 2,420 , neither the simulation time nor its precision is acceptable. On the MATLAB platform, after adopting the tearing method in

TABLE VII
Simulation

| Number of modules | 880/CASE 1 |  | 1210/CASE 2 |  | 2420/CASE 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Error (\%) | Time | Error (\%) | Time | Error (\%) | Time |
| PSIM | 3.4 | 2 hr | 4.5 | 3 hr | N/A | N/A |
| PSIM with tearing | 3.4 | 1 s | 4.5 | 1.1 s | 6.5 | 1.5 s |
| PSpice <br> PSpice | 7.8 | 2.5 hr | N/A | N/A | N/A | N/A |
| with tearing <br> MATLAB | 7.8 | 1.5 hr | 10.8 | 1.7 hr | 14.5 | 2 hr |
| MATLAB with tearing | 3.3 | 2.5 hr | 3.7 | 4.3 hr | N/A | N/A |
|  | 3.3 | 1 hr | 3.7 | 1.2 hr | 8.2 | 1.5 hr |

modeling, the number of modules rose from 1,210 to 2,420 while meeting the required simulation precision of less than $10 \%$. However, its simulation time is too long. Nevertheless, we consider the tearing method as an excellent method that enhances simulation efficiency.

## VII. Conclusions

A PSIM-based macro-model was introduced in this paper. This model features a piecewise linear circuit instead of a nonlinear circuit, resulting in several considerable advantages, such as fast speed, high simulation precision, robust execution, and no convergence issues.

Three types of equivalent macro-models were developed to simplify the modeling of series, parallel, and series-parallel PV modules with uniform irradiance by applying superposition theory. The reason was that the PSIM-based macro-model was a piecewise linear circuit and the $V-I$ characteristics of the PV module had a monotonic function.

The proposed tearing method can divide a large-scale PV array into several small sub-arrays to be modeled, with the involved PV modules receiving uniform irradiance.

The simulated examples of the two large-scale PV arrays were provided to validate the proposed models and methods. In Example 1, a PV array with 440 modules was adopted, and simulation results demonstrated that the simulation efficiency improved by approximately 4,860 times and the simulated precision was maintained at nearly $5 \%$. An actual large-scale PV array with 2,420 modules was set as another example. Simulation results indicated a computing time of nearly 1.5 s and a simulation error of approximately $6.5 \%$. Therefore, the proposed tearing method can help improve simulation efficiency without losing simulation precision.

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