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Control of Variable Reluctance Motors: A Comparison between Classical and Lyapunov-Based Fuzzy Schemes

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ABSTRACT

In this paper, two approaches for designing tracking controllers for a variable reluctance motor (VRM), namely the Lyapunov-based fuzzy approach and the classical approach, are compared The nonlinear model of a VRM is first addressed The two control schemes are introduced afterwards, and then applied to obtain tracking controllers Simulation results of a sample case, to which the methods are applied, are also presented Comparison of the methods based on the results obtained concludes the paper.

Keywords: Fuzzy logic control, Lyapunov methods, variable reluctance motor

1. Introduction

Since the introduction of fuzzy logic, it has found numerous applications in various fields, from social to mathematical studies, and from control to power systems. Although fuzzy systems have usually performed satisfactorily and fuzzy logic has been successfully deployed to tackle several complicated problems, their applicability in the realm of systems control has always been accompanied with a doubt whether or not they can guarantee the stability of the system

Lyapunov-based fuzzy control scheme is a novel approach to systematically design the rule base, while guaranteeing the stability of the closed loop system[1]

These features as well as the smallness of the rule base generated by this approach have made it a suitable candidate in designing stabilizing controllers.

The method is based on a Lyapunov function candidate, it is a function of system states and therefore its input(s) To design a stable closed loop system, the inputs of the system can be manipulated in such a way to make this function satisfy the stability requirements imposed on that Since it is assumed that only partial knowledge about the dynamics of the system is available, it turns out that the requirements can be stated as a number of IF-THEN statements, and hence a fuzzy system

This paper deals with fuzzy and classical control methodologies for a Variable Reluctance Motor (VRM)[2] Mechatronic systems, in general, and VRMs in particular, have been traditionally suitable test beds for both classical and non-classical, e.g fuzzy, control methodologies In addition, VRMs have been among the best candidates in many Adjustable Speed Drive (ASD) applications due to their rugged operating features[3][4].

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Lyapunov-based fuzzy control scheme is applied to design a tracking controller for a VRM; the results are then compared with those obtained from a classical control scheme designed to perform the same task

The nonlinear model of a VRM is first addressed The two control methodologies are described in detail afterwards, and then applied to design tracking controllers. Results of a sample case are also given The performance of the tracking controllers designed using the two methods are compared based on the results obtained.

2. Variable Reluctance Motor Model

A variable reluctance motor can be described by the following state equations[5][6]:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = \sigma_l(y)\theta_l + \Big(\sum_{i=1}^{L} \sigma_i(y)\mathbf{B}^T(u_i)\Big)\boldsymbol{\theta}_u \\ &y = x_1 \end{aligned} \tag{1}$$

where, x_1 and x_2 are the position and the speed of the rotor, respectively u_i is the i-th phase current (the input), and L is the number of phases The term $\sigma_l(y)\theta_l$ represents the load acceleration, and the term $(\sigma_i(y)\mathbf{B}^T(u_i))\boldsymbol{\theta}_u$ represents the acceleration due to the i-th phase current The functions used are defined as follows

$$\begin{aligned} &\sigma_i(y) = \sin\Big(Py - \frac{2\pi(i-1)}{L}\Big) \\ &\mathbf{B}(u_i) = \big[b_1(u_i) \quad b_2(u_i) \quad \cdots \quad b_M(u_i)\big]^T \\ &b_j(u_i) = n\Big(\frac{u_i - (j-1)\delta}{3\delta}\Big) \\ &n(s) = \begin{cases} 0, s < 0, s > 1 \\ 9s^2/2, s \in [0, 1/3] \\ (-18s^2 + 18s - 3)/s, s \in (1/3, 2/3] \\ (3-3s)^2/2, s \in (2/3, 1] \end{cases} \end{aligned} \tag{2}$$

where, P is the number of poles, M is the number of spline intervals and δ is the interval size.

The VRM model is apparently highly nonlinear, the design process will therefore, be quite challenging. The reader is referred to [5] and [6] for a more comprehensive account of VRM modeling.

3. Controller Development Procedures

In this section, the two control strategies mentioned before are addressed. We proceed firstly with Lyapunov-based fuzzy approach and a classical approach will be given afterwards More details of the former can also be found in [1].

3.1 Lyapunov-Based Fuzzy Approach

Let us assume that the system can be described by a set of state equations as follows

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{u}) \\ &\mathbf{y} = \mathbf{h}(\mathbf{x}) \end{aligned} \tag{3}$$

where, **x**, **y** and **u** are state, output and input vectors, respectively Let us also consider a candidate Lyapunov function $V(\mathbf{x})$; the function will indeed be a Lyapunov function if the following conditions are met

$$\begin{aligned} &V(\mathbf{0}) = 0 \\ &V(\mathbf{x}) > 0, \mathbf{x} \neq \mathbf{0} \\ &\dot{V} = \frac{\partial V}{\partial x_i}\dot{x}_i < 0, \mathbf{x} \neq \mathbf{0} \end{aligned} \tag{4}$$

It should be noted that without loss of generality, $\mathbf{x} = \mathbf{0}$ has been assumed to be the equilibrium point

The input(s) can be designed in such a way that V, which is a function of the states and hence the input(s) satisfies the requirements of (4)

To apply the method to the VRM described by (1), we define the tracking error $e = x_l - y_{ref}$ where y_{ref} is the reference trajectory, in addition the candidate Lyapunov function is defined as $V = \frac{1}{2}(e^2 + \dot{e}^2)^2$

Being a quadratic function, V obviously satisfies the first two conditions of (4) Differentiating V yields $\dot{V} = e\dot{e} + \dot{e}w$, where $w = \dot{x}_2 - \ddot{y}_{ref}$. In order for V to become a true Lyapunov function, we need to have $\dot{V} = e\dot{e} + \dot{e}w < 0$ The following rule base is formulated in such a way to force V to satisfy this last requirement·

- IF e is *positive* and $\dot{e}$ is *positive*, THEN w is *negative big*.

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- IF e is *negative* and $\dot{e}$ is *negative*, THEN w is *positive big*
- IF e is *positive* and $\dot{e}$ is *negative*, THEN w is *zero*.
- IF e is *negative* and $\dot{e}$ is *positive*, THEN w is *zero*.

The above conditions should be used to determine the necessary current in each phase To derive the phase currents, we note that:

$$\dot{x}_2 = w + \ddot{y}_{ref} = \sigma_l(y)\theta_l + (\sum_{i=1}^{L} \sigma_i(y)\mathbf{B}^T(u_i))\theta_u \qquad (5)$$

Solving this nonlinear equation yields the phase currents. Taking into account the highly nonlinear nature of the functions involved, the solution can be quite challenging and needs a highly tailored method. For this purpose the approach presented in [7] is adopted The approach involves the steps of determining the phase(s) to be excited and finding the phase current(s) needed. The details of the method are beyond the scope of this paper and will therefore be avoided

3.2 Classical Control Approach

A classical approach to the design of tracking controller for a VRM can be formulated as follows:

Define the auxiliary signal

$$v = c_1(y_{ref} - x_1) + c_2(\dot{y}_{ref} - x_2) + \ddot{y}_{ref} \qquad (6)$$

where, c_1 and c_2 are constant gains, which are selected so that $s^2+c_2s+c_1$ is Hurwitz Equating the left hand side of the relation for $\dot{x}_2$ with v yields:

$$\sigma_l(y)\theta_l + (\sum_{i=1}^{L} \sigma_i(y)\mathbf{B}^T(u_i))\boldsymbol{\theta}_u = v \qquad (7)$$

Similar to the previous section, this equation is solved to obtain phase currents u_i. The solution method follows exactly the same steps.

It is also interesting to note the dynamics of the error. Let us define $e_1 = x_1 - y_{ref}$ and $e_2 = x_2 - \dot{y}_{ref}$.

Then, we have.

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -c_1 & -c_2 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \qquad (8)$$

which, under the assumptions made about c_1 and c_2, will cause the tracking error and its derivative to converge to zero. It is also observed that proper selection of the constants c_1 and c_2 directly affects the rate of convergence of the e_1 and e_2

In the following section, the two aforementioned methods are applied to a sample VRM, and the results are compared.

4. Simulation Results

This chapter is devoted to computer simulation results of a sample motor. Tracking controllers for the motor are designed using the two previously mentioned methods. Performances of the controllers obtained are then compared taking into account issues such as robustness to parameter deviation in VRM model

Parameters of the VRM are given in Table 1. Reference trajectory is considered to be $y_{ref} = \pi - \pi\cos(\pi t)$, which causes the rotor to smoothly rotate between 0 and 2π *rad* with a frequency of 0.5 Hz

Before proceeding with the simulation results, it should be asserted that it is a common practice not to measure x_2, but to estimate it through approximate differentiation This is the underlying idea of sensorless motor drive applications[7]. The estimation can be formulated as follows:

$$\hat{x}_2 = \frac{w_0^2 s}{s^2 + 1.4 w_0 s + w_0^2} x_1 \qquad (9)$$

where, $\hat{x}_2$ is the estimated value of x_2 The cutoff frequency w_0 is chosen to be by far larger than the highest frequency component of the system. In the following simulations, the value of w_0 is assigned to be 150 *rad/sec* Rotor position samples are taken with a sampling frequency of 400 Hz

The membership functions for the linguistic variables in the fuzzy inference system of chapter 3 1 are taken to be of the following forms.

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Table 1 Parameters of the VRM

Parameter	Value
Number of spline functions (M)	5
Spline interval size (δ)	0 25
Load coefficient (θ_l)	67
Winding coefficients (θ_i)	$50i$
Number of poles (P)	8
Number of phases (L)	3

$$\begin{aligned} &\mu_{positive}(x) = \frac{1}{1+\exp(-30x)} \\ &\mu_{negative}(x) = \frac{1}{1+\exp(30x)} \\ &\mu_{negative-big}(u) = \exp(-(u+5)^2) \\ &\mu_{zero}(u) = \exp(-u^2) \\ &\mu_{positive-big}(u) = \exp(-(u-5)^2) \end{aligned} \qquad (10)$$

The fuzzy inference system deploys the product inference engine and the center of area method for defuzzification.

Design parameters for the classical approach are initially set to $c_1 = 6$ and $c_2 = 5$ Fig. 1 shows the reference trajectory and the actual rotor positions obtained using fuzzy and classical approaches. Initial conditions for the states are $x_1(0) = 30\ deg$ and $x_2(0) = 0\ 6\ rad/sec$

Fig. 2 shows the tracking errors for the same conditions. Tracking error in steady state is also shown in Fig 3. It is observed that the fuzzy controller has performed successfully, and has caused the rotor to follow the reference trajectory within a reasonable time. The reason for faster response of the classical controller is the fact that c_1 and c_2 are chosen so that the error is damped quite rapidly Manipulating c_1 and c_2 can obviously result in faster or slower convergence rates Phase currents for the classical and fuzzy methods are shown in Figs 4 and 5 It is clear that at every instance, a proper combination of phases with certain amounts of currents flowing through them has been selected to provide the electrical torque needed to rotate the rotor

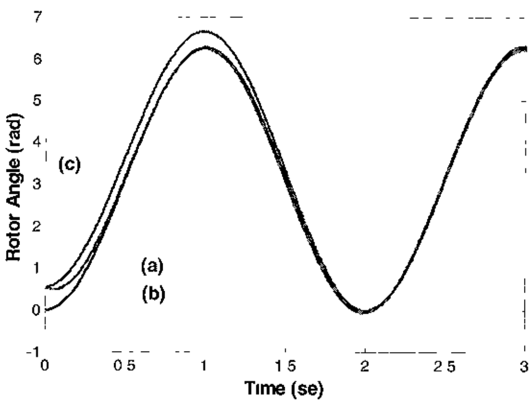

Fig 1 Rotor Position (a) Reference Trajectory, (b) Actual Rotor Position (Classical Method), (c) Actual Rotor Position (Fuzzy Method)

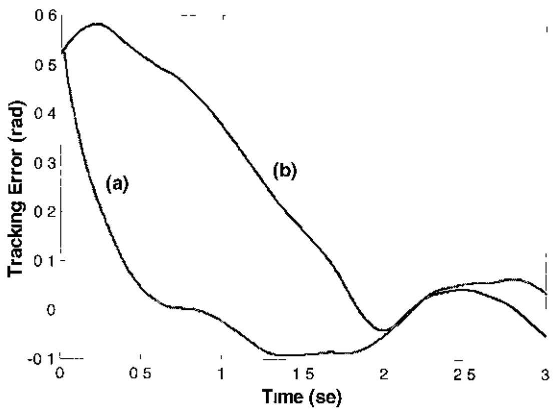

Fig 2 Tracking Error (a) Classical Controller, (b) Fuzzy Controller

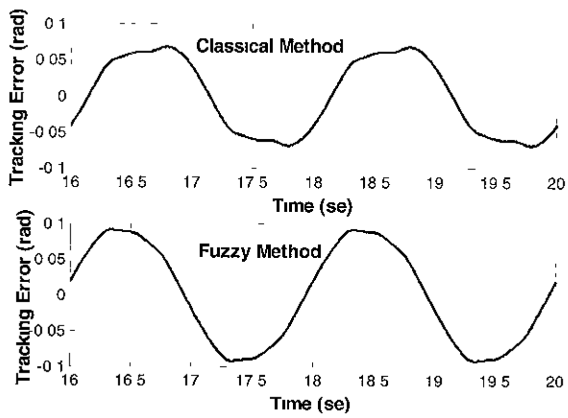

Fig 3 Steady State Tracking Error

It is interesting to note the robustness of the performance of these two control schemes. To do so, two strategies are adopted, one to investigate robustness to model parameter deviations, and the other to study the effect of measurement noise on the performance of the system.

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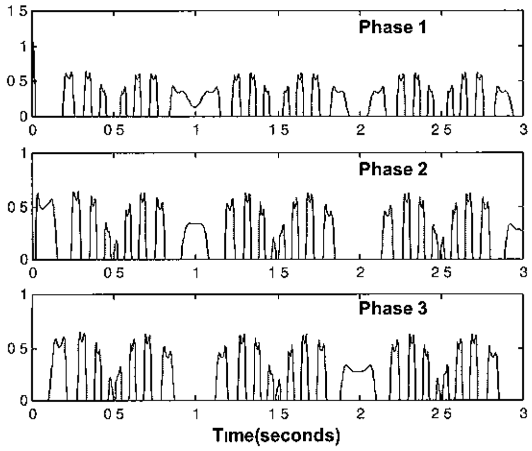

Fig 4 Phase Currents (A) for the Classical Case

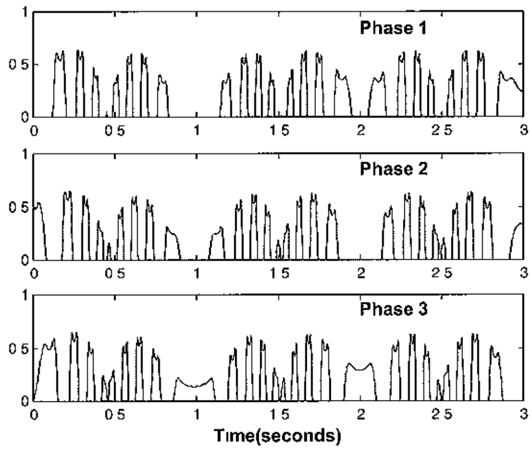

Fig 5 Phase Currents (A) for the Fuzzy Case

Table 2 shows the perturbed values of load and winding coefficients, which show a quite significant change. Note that with reference to the (1), any increase in winding coefficients causes the nonlinear part to be more pronounced, the perturbations introduced in the parameters therefore cause further nonlinearity in the system Other parameters are assumed to be unchanged

The reference and actual rotor positions for the two controllers are shown in Fig 6. Initial conditions are the same as before Although the parameter deviations are quite considerable, the controllers have successfully performed. This is a promising sign of their performance robustness Transient and steady state tracking error for the two controllers are also shown in Figs. 7 and 8, respectively.

Table 2 Perturbed Values of VRM Model Parameters

Parameter	Value
Load coefficient (θ_l)	17
Winding coefficients (θ_i)	100i

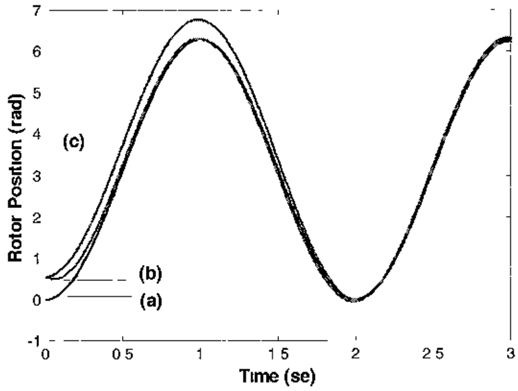

Fig 6 Rotor Position (a) Reference Trajectory, (b) Actual Rotor Position (Classical Method), (c) Actual Rotor Position (Fuzzy Method)

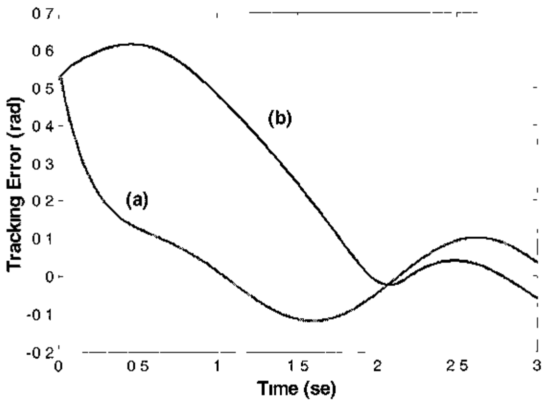

Fig 7 Tracking Error. (a) Classical Controller, (b) Fuzzy Controller

To investigate the effect of measurement noise on the performance of the control system, a uniformly distributed noise over the interval [-0.05, 0.05] was added to the x_l measurement Figs. 9 and 10 show the tracking performance of the system under such conditions for the classical and fuzzy schemes, respectively Reference trajectory and initial conditions are left unchanged Model parameters were also set back to their original values listed in Table 1

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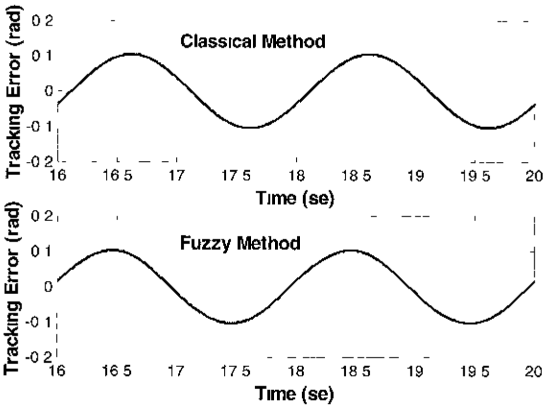

Fig. 8 Steady State Tracking Error

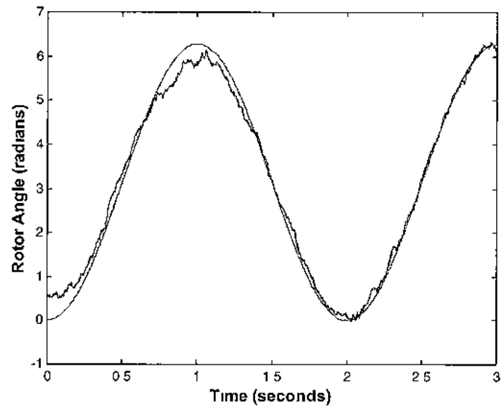

Fig 9 Tracking Performance under Measurement Noise (Classical Controller)

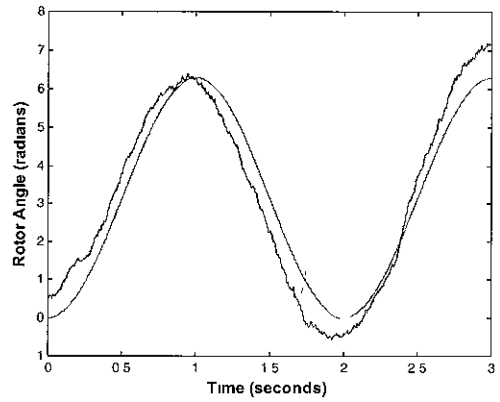

Fig 10 Tracking Performance under Measurement Noise (Fuzzy Controller)

It is seen that the classical controller has a better performance with regard to disturbance rejection. This is partly due to the fact the tracking error dynamics of the classical approach can be adequately determined by the coefficients c_1 and c_2. It should be noted that manipulating the parameters of the fuzzy inference engine, i e membership function parameters, may result in better disturbance rejection index for the fuzzy scheme.

5. Conclusions

Lyapunov-based fuzzy control scheme for a VRM has been compared with a classical control approach

Lyapunov-based fuzzy approach not only guarantees the stability of the closed loop system, but also results in a very small rule base The simulation results for this method show close conformity with those of a classical approach. The results also show robustness of the method to parameter deviations However, the disturbance rejection of the fuzzy system may be inferior than that of the classical controller, further adjustment of the inference engine parameters may be needed to overcome this deficiency The method can therefore stand out as a prime candidate in controller synthesis for nonlinear systems

The most appealing features of the approach are its small rule base and its applicability to a wide range of nonlinear systems Taking into account the wide range of nonlinearity forms that can happen in a real system, Lyapunov-based fuzzy scheme can significantly ease the controller design procedure, with limited knowledge of the nonlinearity type

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