JPE 16-5-19

http://dx.doi.org/10.6113/JPE.2016.16.5.1802
ISSN(Print): 1598-2092 / ISSN(Online): 2093-4718

A Cross Regulation Analysis for Single-Inductor Dual-Output CCM Buck Converters

Yao Wang†, Jianping Xu*, and Guohua Zhou*

†School of Electrical and Information Engineering, Southwest University for Nationalities, Chengdu, China
*School of Electrical Engineering, Southwest Jiaotong University, Chengdu, China

Abstract

Cross regulation is a key technical issue of single-inductor multiple-output (SIMO) DC-DC converters. This paper investigates the cross regulation in single-inductor dual-output (SIDO) Buck converters with continuous conduction mode (CCM) operation. The expressions of the DC voltage gain, control to the output transfer function, cross regulation transfer function, cross coupled transfer function and impedance transfer function of the converter are presented by the time averaging equivalent circuit approach. A small signal model of a SIDO CCM Buck converter is built to analyze this cross regulation. The laws of cross regulation with respect to various load conditions are investigated. Simulation and experiment results verify the theoretical analysis. This study will be helpful for converter design to reduce the cross regulation. In addition, a control strategy to reduce cross regulation is performed.

Key words: CCM buck converter, Cross regulation, SIDO, Small signal model

I. INTRODUCTION

These days, portable devices such as digital assistants and mobile phones require multiple power supplies with different output voltages [1]-[3]. A single-inductor multiple-output (SIMO) DC-DC converter with only one inductor is a good candidate for providing multiple outputs. It has attracted a lot of attention by the researchers due to its advantages of low cost, high efficiency and small volume size [4]-[6].

According to difference in the inductor current operation mode, a single-inductor dual-output (SIDO) DC-DC converter has three operation modes including the inductor current discontinuous conduction mode (DCM), the pseudo-continuous conduction mode (PCCM) and the continuous conduction mode (CCM). Time multiplexing control techniques in the DCM are used in [6] and [7]. Although these approaches can eliminate cross regulation under light loads, a large inductor current and output voltage ripple exist under heavy loads. To overcome this large ripple, a SIDO PCCM DC-DC converter was proposed in [8]. However, an additional freewheeling switch results in power loss during the freewheeling stage [9]-[10].

Compared with the SIDO DCM and PCCM DC-DC converters, the SIDO CCM DC-DC converter has the advantages of a low ripple and less power loss due to it not having a zero inductor current stage or a freewheeling stage. However, when one SIDO CCM DC-DC converter load changes, the other output voltage changes. This is refered to as cross regulation and it plays a major role in the performance of SIDO converters [11]-[13]. Cross regulation influences the normal work of a converter and can even cause system instability. In spite of this, the SIDO CCM DC-DC converter has been researched due to its numerous advantages. Therefore, it is important to research the cross regulation of the SIDO CCM DC-DC converter. Control techniques for suppressing the cross regulation of SIDO CCM Buck converters was researched in [14]-[17]. However, a theoretical analysis of cross regulation is lacking.

In this paper, the cross regulation of SIDO CCM Buck converters is studied. According to the time averaging equivalent circuit approach [18], a small signal model to analysis the cross regulation of a SIDO CCM Buck converter is proposed. Based on this, a small signal model of a voltage controlled SIDO CCM Buck converter is established to analyze the law of cross regulation with the load changed. The analysis results can be used for the designing of the converter load condition to reduce cross regulation.

This paper is organized as follows: Section II illustrates the

Manuscript received Nov. 20, 2015; accepted Apr. 30, 2016
Recommended for publication by Associate Editor Se-Kyo Chung.
†Corresponding Author: wy886@live.com
Tel: +86-28-85928384, Southwest University for Nationalities
*School of Electrical and Information Engineering School, Southwest Jiaotong University, China

© 2016 KIPE

SIDO CCM Buck converter including the operation principle and small signal model. The small signal model of a voltage controlled SIDO CCM Buck converter is established in Section III. Cross regulation analyses with different load parameters are performed in Section IV. A control strategy to reduce cross regulation is provided in Section V. Simulation and experiment verification is performed in Section VI. A conclusion is provided in Section VII.

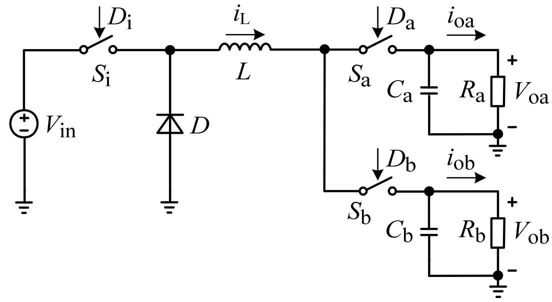


Fig. 1. Circuit diagram of SIDO CCM Buck converter.

II. SIDO CCM BUCK CONVERTER

A. Operation Principle

Fig. 1 shows a circuit diagram of a SIDO CCM Buck converter with an output A and an output B. It consists of one inductor L, two output capacitors C_a and C_b, one input voltage source V_{in}, and two outputs with voltages V_{oa} and V_{ob}. The power switch S_i and freewheeling diode D control the input power, and the power switches S_a and S_b are used to control the power distribution from the input to the two outputs. The output voltages V_{oa} and V_{ob} are regulated by adjusting the duty cycles D_i, D_a and D_b. When a SIDO Buck converter operates in CCM, there is D_a+D_b=1, *i.e.* control pulses of output A and output B are complementary.

There are three operation timings due to the duty cycles D_i>D_b, D_i=D_b and D_i<D_b [19]. Fig. 2 shows the steady-state timing and operation mode of a SIDO CCM Buck converter under D_i<D_b. In this condition, there are three operation modes in a switching cycle, as shown in Fig. 2 (b).

Mode I: The power switches S_i and S_b are turned on, S_a and the freewheeling diode D are turned off, the inductor current i_L increases to the peak current I_{L1} with a slope of (V_{in}-V_{ob})/L.

Mode II: S_i is turned off, D is turned on, S_a is kept off and S_b is kept on, while i_L decrease to I_{L2} with a slope of -V_{ob}/L.

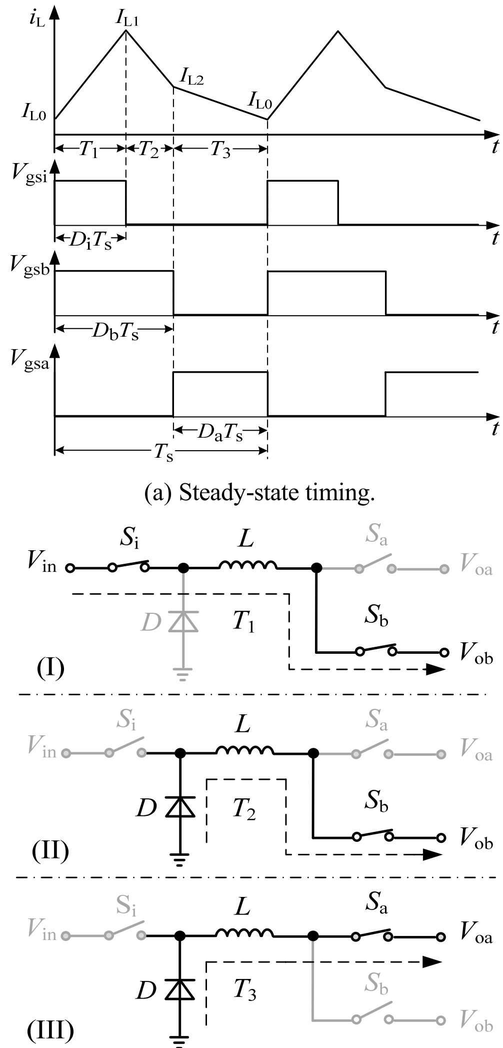
Mode III: S_i is kept off and D is kept on, S_a is turned on and S_b is turned off, and i_L decreases with a slope of -V_{oa}/L until the beginning of next switching cycle.

Based on time averaging equivalent circuit approach, the DC voltage gains of output A and output B are as:

$$M_a = \frac{D_i(1-D_b)R_a}{D_b^2 R_b + (1-D_b)^2 R_a} \tag{1}$$

$$M_b = \frac{D_i D_b R_b}{D_b^2 R_b + (1-D_b)^2 R_a} \tag{2}$$

As shown in (1) and (2), the DC voltage gains of a SIDO CCM Buck converter are associated with D_i and D_b. The output voltages V_{oa} and V_{ob} can be regulated by adjusting D_i and D_b. In addition, the DC voltage gains of a SIDO CCM Buck converter are associated with R_a and R_b. The variation of one output will affect the other output, *i.e.* cross regulation exists in this converter.



(b) Operation mode.

Fig. 2. Steady-state timing and operation mode of SIDO CCM Buck converter.

B. Small Signal Modeling for a SIDO CCM Buck Converter

According to the time averaging equivalent circuit approach, S_i and S_a can be replaced by the controlled voltage sources $\hat{v}_{si}(s)$ and $\hat{v}_{sa}(s)$, while D and S_b can be replaced by the controlled current source $\hat{i}_D(s)$ and $\hat{i}_{sb}(s)$. Then the time averaging AC small singal equivalent circuit can be given in Fig. 3, where $\hat{i}_D(s)$, $\hat{v}_{si}(s)$, $\hat{i}_{sb}(s)$, and $\hat{v}_{sa}(s)$ are given by:

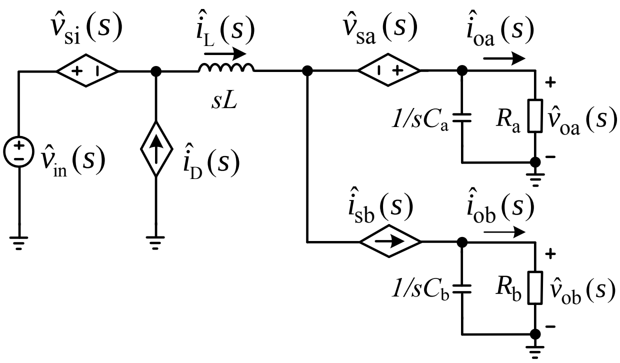


Fig. 3. AC small signal equivalent circuit of SIDO CCM Buck converter.

$$\begin{cases} \hat{i}_D(s) = (1-D_i)\hat{i}_L(s) - \hat{d}_i(s)I_L \\ \hat{v}_{si}(s) = (1-D_i)\hat{v}_{in}(s) - \hat{d}_i(s)V_{in} \\ \hat{i}_{sb}(s) = D_b\hat{i}_L(s) + \hat{d}_b(s)I_L \\ \hat{v}_{sa}(s) = D_b(\hat{v}_{oa}(s) - \hat{v}_{ob}(s)) + \hat{d}_b(s)(V_{oa} - V_{ob}) \end{cases} \tag{3}$$

Where $\hat{i}_L(s)$, $\hat{d}_i(s)$ and $\hat{d}_b(s)$ are the small signal disturbance variables of the inductor current i_L, D_i and D_b, while $\hat{v}_{in}(s)$, $\hat{v}_{oa}(s)$ and $\hat{v}_{ob}(s)$ are the small signal disturbance variables of V_{in}, V_{oa} and V_{ob}, respectively.

From the AC small signal equivalent circuit of a SIDO CCM Buck converter, as shown in Fig. 3, it can have:

$$\hat{i}_L(s) = \frac{\hat{v}_{oa}(s)}{R_{eqa}(s)} + \frac{\hat{v}_{ob}(s)}{R_{eqb}(s)} \tag{4}$$

$$\hat{v}_{in}(s) = \hat{v}_{si}(s) + sL\hat{i}_L(s) - \hat{v}_{sa}(s) + \hat{v}_{oa}(s) \tag{5}$$

where $R_{eqa}(s) = 1/(\frac{1}{R_a} + sC_a)$ and $R_{eqb}(s) = 1/(\frac{1}{R_b} + sC_b)$.

From (3) to (5), the control to output transfer functions $G_{11}(s)$ and $G_{22}(s)$ of output A and output B are:

$$G_{11}(s) = \left.\frac{\hat{v}_{oa}(s)}{\hat{d}_i(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_b(s)=0} = \frac{V_{in}(1-D_b)R_{eqa}(s)}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + SL} \tag{6}$$

$$G_{22}(s) = \left.\frac{\hat{v}_{ob}(s)}{\hat{d}_b(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_i(s)=0} = \frac{I_LR_{eqb}(s)[sL + (1-D_b)R_{eqa}(s)] + D_bR_{eqb}(s)(V_{oa} - V_{ob})}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{7}$$

The cross coupled transfer functions $G_{21}(s)$ and $G_{12}(s)$ of output A and output B are:

$$G_{21}(s) = \left.\frac{\hat{v}_{oa}(s)}{\hat{d}_b(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_i(s)=0} = \frac{-I_LR_{eqa}(s)(sL + D_bR_{eqb}(s)) + (1-D_b)R_{eqa}(s)(V_{oa} - V_{ob})}{D_b^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{8}$$

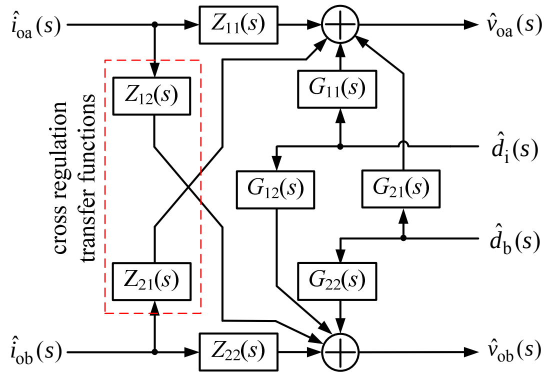


Fig. 4. The small signal model of SIDO CCM Buck converter.

$$G_{12}(s) = \left.\frac{\hat{v}_{ob}(s)}{\hat{d}_i(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_b(s)=0} = \frac{V_{in}D_bR_{eqb}(s)}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{9}$$

The output impedances $Z_{11}(s)$ and $Z_{22}(s)$ of output A and output B are:

$$Z_{11}(s) = \left.\frac{\hat{v}_{oa}(s)}{\hat{i}_{oa}(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_i(s)=0, \hat{d}_b(s)=0} = \frac{D_b^2R_{eqa}(s)R_{eqb}(s) + sLR_{eqa}(s)}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{10}$$

$$Z_{22}(s) = \left.\frac{\hat{v}_{ob}(s)}{\hat{i}_{ob}(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_i(s)=0, \hat{d}_b(s)=0} = \frac{D_a{}^2R_{eqb}(s)R_{eqa}(s) + sLR_{eqb}(s)}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{11}$$

The cross regulation transfer functions, $Z_{21}(s)$ of output B to output A and $Z_{12}(s)$ of output A to output B, are:

$$Z_{21}(s) = \left.\frac{\hat{v}_{oa}(s)}{\hat{i}_{ob}(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_i(s)=0, \hat{d}_b(s)=0} = \frac{-D_bD_aR_{eqa}(s)R_{eqb}(s)}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{12a}$$

$$Z_{12}(s) = \left.\frac{\hat{v}_{ob}(s)}{\hat{i}_{oa}(s)}\right|_{\hat{v}_{in}(s)=0, \hat{d}_i(s)=0, \hat{d}_b(s)=0} = \frac{-D_bD_aR_{eqa}(s)R_{eqb}(s)}{D_b{}^2R_{eqb}(s) + (1-D_b)^2R_{eqa}(s) + sL} \tag{12b}$$

From (12), it can be seen that $Z_{21}(s) = Z_{12}(s)$.

Select the currents of the two outputs in a SIDO CCM Buck converter as system inputs. Similarly, select the voltages of the two outputs in a SIDO CCM Buck converter as system outputs. Thus, a SIDO CCM Buck converter can be equivalent to a dual-input and dual-output system. According to the transfer functions from (6) to (12), the small signal model of a SIDO CCM Buck converter is established as shown in Fig. 4. This is used to analyze the cross regulation between the two outputs of the converter in this paper.

As shown in Fig. 4, the output current $\hat{i}_{oa}(s)$ of output A affects the output voltage $\hat{v}_{ob}(s)$ of output B through the cross regulation transfer function $Z_{12}(s)$, and the output current $\hat{i}_{ob}(s)$ of output B affects the output voltage $\hat{v}_{oa}(s)$ of output A through the cross regulation transfer function $Z_{21}(s)$.

III. MODELING OF A VOLTAGE CONTROLLED SIDO CCM BUCK CONVERTER

There are two outputs in a SIDO CCM Buck converter. Therefore, at least two control loops are required to realize the control of a SIDO CCM Buck converter. For the small signal model of the SIDO CCM Buck converter shown in Fig. 4, two PI compensation functions $F_{11}(s)$, $F_{22}(s)$ and two PWM transfer functions $G_{m1}(s)$, $G_{m2}(s)$ are introduced from the outputs to keep the output voltage at their order values. Thus, the small signal model of a voltage controlled SIDO CCM Buck converter can be obtained, as shown in Fig. 5(a). The circuit diagram of the voltage controlled SIDO CCM Buck converter is shown in Fig. 5(b). As shown in Fig. 5(b), the feedback voltage V_{oa} determines the duty cycle D_i of switch S_i, and feedback voltage V_{ob} determines the duty cycle D_b of switch S_b. Since $D_a+D_b=1$, the duty cycle D_a of switch S_a is also determined by V_{ob}.

$F_{11}(s)$, $F_{22}(s)$ and $G_{m1}(s)$, $G_{m2}(s)$ are:

$$F_{11}(s)=K_{p1}+\frac{K_{I1}}{s}, \qquad F_{22}(s)=K_{p2}+\frac{K_{I2}}{s} \tag{13}$$

$$G_{m1}(s)=\frac{1}{V_{m1}}, \qquad G_{m2}(s)=\frac{1}{V_{m2}} \tag{14}$$

To simplify the analysis, assume that $V_{m1} = V_{m2} = 1$ in (14). Thus, the transfer function $G_{m1}(s) = G_{m2}(s) = 1$. According to Fig. 5(a), the output impedances $Z'_{11}(s)$ of output A and $Z'_{22}(s)$ of output B are:

$$Z'_{11}(s)=\frac{\hat{v}_{oa}(s)}{\hat{i}_{oa}(s)}\approx\frac{Z_{11}(s)}{G_{11}(s)F_{11}(s)} \tag{15}$$

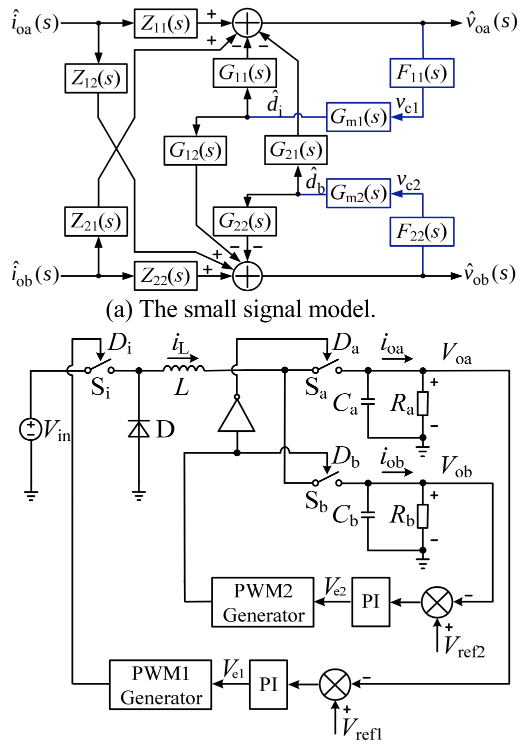
$$Z'_{22}(s)=\frac{\hat{v}_{ob}(s)}{\hat{i}_{ob}(s)}\approx\frac{Z_{22}(s)}{G_{22}(s)F_{22}(s)} \tag{16}$$

The cross regulation transfer functions $Z'_{21}(s)$ of output B to output A and $Z'_{12}(s)$ of output A to output B are:

$$Z'_{21}(s)=\frac{\hat{v}_{oa}(s)}{\hat{i}_{ob}(s)}\approx\frac{Z_{21}(s)G_{22}(s)-Z_{22}(s)G_{21}(s)}{(G_{12}(s)G_{21}(s)-G_{11}(s)G_{22}(s))F_{11}(s)} \tag{17}$$

$$Z'_{12}(s)=\frac{\hat{v}_{ob}(s)}{\hat{i}_{oa}(s)}\approx\frac{Z_{12}(s)G_{11}(s)-Z_{11}(s)G_{12}(s)}{(G_{12}(s)G_{21}(s)-G_{11}(s)G_{22}(s))F_{22}(s)} \tag{18}$$

Based on (17) and (18), the cross regulation of the voltage controlled SIDO CCM Buck converter can be analyzed.



(a) The small signal model.

(b) The circuit diagram.

Fig. 5. Voltage controlled SIDO CCM Buck converter.

IV. CROSS REGULATION ANALYSIS OF THE CONVERTER

A. Frequency Domain Analysis

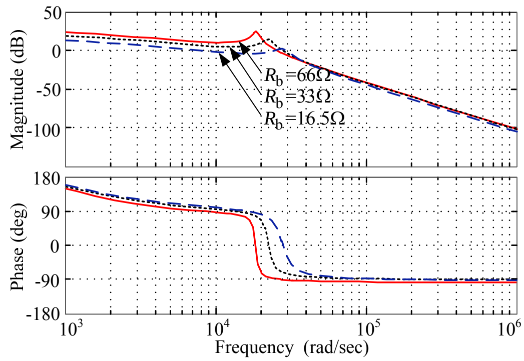
1) Cross Regulation of Output B to Output A with Different R_b or of Output A to Output B with Different R_a.: Fig. 6(a) gives the transfer function $Z'_{21}(s)$ with different R_b. This shows that the low frequency gain of the transfer function $Z'_{21}(s)$ decreases with a decrease of R_b, *i.e.* the cross regulation of output B to output A decreases gradually with a decrease of R_b.

Fig. 6(b) gives the transfer function $Z'_{12}(s)$ with different R_a. This shows that the low frequency gain of the transfer function $Z'_{12}(s)$ decreases with a decrease of R_a, *i.e.* the cross regulation of output A to output B decreases gradually with a decrease of R_a.

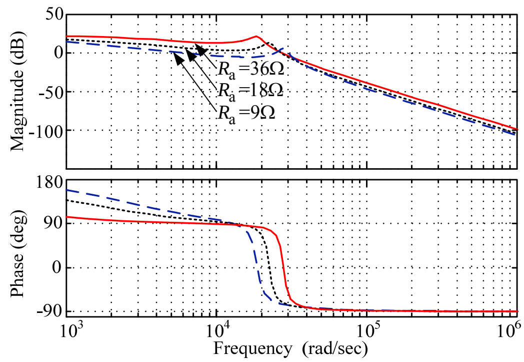
From the frequency domain analysis shown in Fig. 6, it can be concluded that a heavier cross regulation from output A to output B occurs when the load of output A is lighter. It is same for the cross regulation from output B to output A.

2) Cross Regulation of Output A to Output B with Different R_b or of Output B to Output A with Different R_a.: The cross regulation of output A to output B with different R_b and of output B to output A with different R_a are investigated in this section.

Fig. 7(a) gives the cross transfer function $Z'_{21}(s)$ with different R_a. This shows that the low frequency gain of the

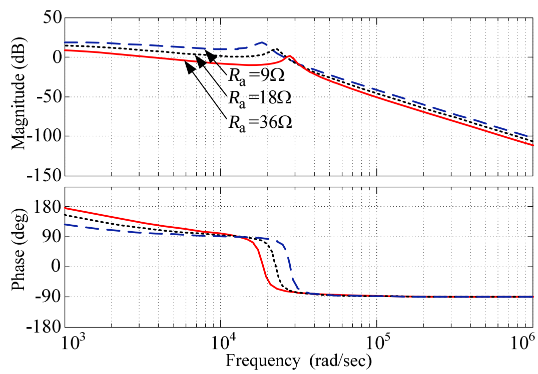


(a) The Bode plot of transfer function $Z'_{21}(s)$ with different R_b.

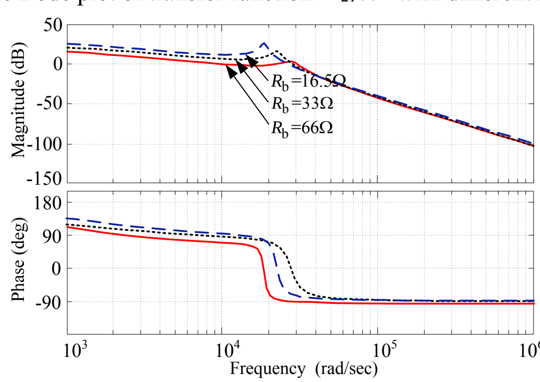


(b) The Bode plot of transfer function $Z'_{12}(s)$ with different R_a.

Fig. 6. The cross regulation of one output with different parameters to another output.



(a) The Bode plot of transfer function $Z'_{21}(s)$ with different R_a.



(b) The Bode plot of transfer function $Z'_{12}(s)$ with different R_b.

Fig. 7. The cross regulation of one output to the another output with different parameters.

transfer function $Z'_{21}(s)$ decreases with an increase of R_a, *i.e.* the affection of output B to output A decreases gradually with an increase of R_a. Fig. 7(b) shows the cross transfer function $Z'_{12}(s)$ with different R_b. This shows that the low frequency gain of the transfer function $Z'_{12}(s)$ decreases with an increase of R_b, *i.e.* the affection of output A to output B decreases with an increase of R_b.

The frequency domain analysis results shown in Fig. 6 and Fig. 7 show that a heavier cross regulation from output B to output A occurs when output B has a lighter load condition and when output A has a heavier load condition. Likewise, a weaker cross regulation from output B to output A occurs when output B has a heavier load condition and when output A has a lighter load condition. Thus, when the load change is in output B, to reduce the cross regulation to output A, output B should be designed with a heavier load condition and output A should be designed with a lighter load condition.

V. Cross Regulation Reduction Control Strategy

In this section, a control strategy to reduce cross regulation is proposed based on the small signal model shown in Fig. 4(a). The voltages of output A and output B can be expressed as:

$$v_{oa}(s) = i_{oa}(s)\cdot Z'_{11}(s) + i_{ob}(s)\cdot Z'_{21}(s) \tag{19}$$

$$v_{ob}(s) = i_{ob}(s)\cdot Z'_{22}(s) + i_{oa}(s)\cdot Z'_{12}(s) \tag{20}$$

It can be seen that cross regulation can be reduced by adding output current compensation functions.

Based on a small signal equivalent model of a voltage control SIDO CCM Buck converter, the output current compensation functions $Z_1(s)$ and $Z_2(s)$, as shown in Fig. 8, are introduced to reduce cross regulation. With these output current compensation functions, the output voltage can be represented by:

$$\begin{aligned} V_{oa}(s) &= i_{oa}(s)\cdot Z'_{11}(s) + i_{ob}(s)\cdot Z'_{21}(s) \\ &\quad + i_{ob}(s)\cdot Z_2(s)\cdot\frac{G_{11}(s)}{1-G_{11}(s)F_{11}(s)} \\ &= i_{oa}(s)\cdot Z'_{11}(s) \end{aligned} \tag{21}$$

$$\begin{aligned} V_{ob}(s) &= i_{ob}(s)\cdot Z'_{22}(s) + i_{oa}(s)\cdot Z'_{12}(s) \\ &\quad + i_{oa}(s)\cdot Z_1(s)\cdot\frac{G_{22}(s)}{1-G_{22}(s)F_{22}(s)} \\ &= i_{ob}(s)\cdot Z'_{22}(s) \end{aligned} \tag{22}$$

with:

$$Z_1(s) = Z'_{12}(s)\cdot\frac{G_{22}(s)F_{22}(s)-1}{G_{22}(s)} \tag{23}$$

$$Z_2(s) = Z'_{21}(s)\cdot\frac{G_{11}(s)F_{11}(s)-1}{G_{11}(s)} \tag{24}$$

Theoretically, cross regulation can be eliminated by

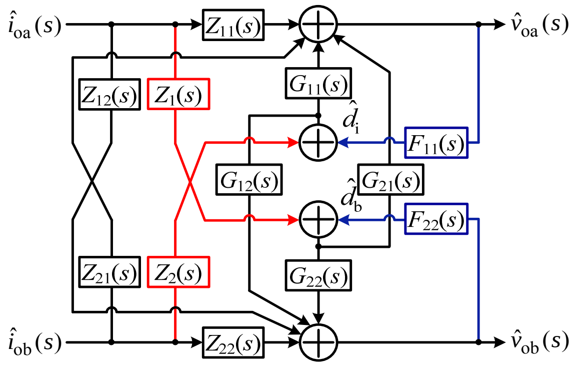


Fig. 8. The circuit schematic and small signal model of output current control SIDO Buck converter.

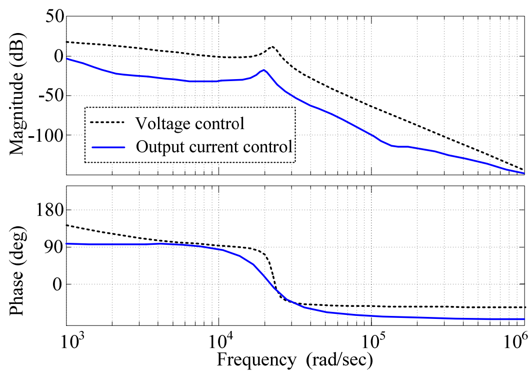


Fig. 9. Bode plot of voltage control and output current control SIDO CCM Buck converter for V_{oa} with I_{ob} changed.

introducing Z_1(s) and Z_2(s). However, the derivation process involves some approximations. Therefore, cross regulation is not perfectly eliminated but it is greatly suppressed.

Fig. 9 depicts the frequency response for the cross regulation of output B to output A. The dashed line and solid line represent a voltage controlled and an output current controlled SIDO CCM Buck converter, respectively. It can be seen that the low frequency gain of the output current controlled SIDO CCM Buck converter is lower than that of the voltage control. This means that the cross regulation of the output current controlled SIDO CCM Buck converter is smaller.

VI. Simulation and Experiment Results

A. Frequency Domain Analysis

MATLAB/Simulink based simulations of a SIDO CCM Buck converter with the circuit parameters listed in Table I are presented in this section to investigate the analysis results.

1) Cross Regulation of Output B to Output A with Different R_b or of Output A to Output B with Different R_a.: R_b=66Ω and 33Ω correspond to the output currents I_{ob} = 50mA and 100mA, respectively. R_a=36Ω and 18Ω correspond to the output currents I_{oa}=50mA and 100mA, respectively.

TABLE I
CIRCUIT PARAMETERS OF SIDO CCM BUCK CONVERTER

Variable	Definition	Value
V_{in}	Input voltage	10V
R_a	Rated load resistor of output A	18Ω
V_{oa}	Voltage of output A	1.8V
R_b	Rated load resistor of output B	33Ω
V_{ob}	Voltage of output B	3.3V
L	Inductor	60uH
C_a	Capacitor of output A	220µF
C_b	Capacitor of output B	220µF
T_s	Switching period	10µs

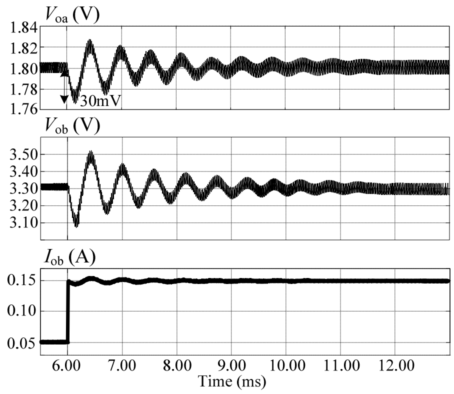
Fig. 10(a) shows the transient waveform when I_{ob} is step changed from 50mA to 100mA. The cross regulation of output B to output A is 30mV. Fig. 10(b) shows the transient waveform when I_{ob} is step changed from 100mA to 200mA. The cross regulation of output B to output A is 20mV. From Fig. 10(a) and Fig. 10(b), it can be seen that a heavier cross regulation of output B to output A occurs when I_{ob} is step changed from 50mA to 100mA than when it is step changed from 100mA to 200mA. *i.e.* heavier cross regulation from output B to output A occurs at a lighter load of output B.

Fig. 10(c) shows the transient waveform when I_{oa} is step changed from 50mA to 100mA. The cross regulation of output A to output B is 220mV. Fig. 10(d) shows the transient waveform when I_{oa} is step changed from 100mA to 200mA. The cross regulation of output A to output B is 190mV. From Fig. 10(c) and Fig. 10(d) it can be seen that a heavier cross regulation of output A to output B occurs when I_{oa} is step changed from 50mA to 100mA than when it is step changed from 100mA to 200mA, *i.e.* heavier cross regulation from output A to output B occur at a lighter load of output A.

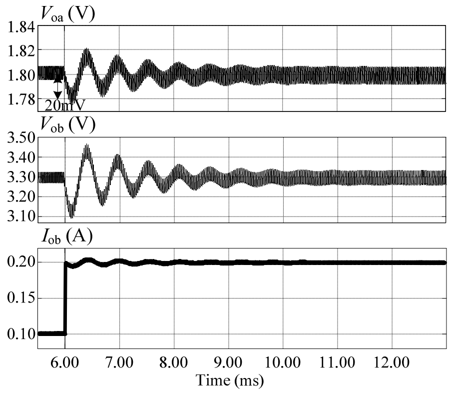
2) Cross regulation of output A to output B with different R_b or of output B to output A with different R_a.: R_b=33Ω and 16.5Ω correspond to the output currents I_{ob}=100mA and 200mA, respectively. R_a=18Ω and 9Ω correspond to the output currents I_{oa}=100mA and 200mA, respectively.

A transient state simulation waveform when I_{oa} is equal to 100mA and I_{ob} is step changed from 100mA to 200mA is shown in Fig. 11(a). It is shown that the cross regulation of output B to output A is 20mV under this condition. The transient state simulation waveform when I_{oa} is equal to 200mA and I_{ob} is step changed from 100mA to 200mA is shown in Fig. 11(b). As shown in Fig. 11(b), the cross regulation of output B to output A is 60mV. Thus, it is verified that when output A operates at a lighter load, a weaker cross regulation of output B to output A is produced. In addition, when output A operates at a heavier load, a greater cross regulation of output B to output A is produced.

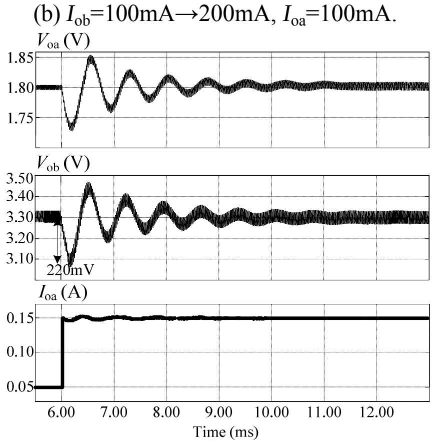
The transient state simulation waveform when I_{ob} is equal to 100mA and I_{oa} is step changed from 100mA to 200mA is shown in Fig. 11(c). It can be seen that the cross regulation of output A to output B is 170mV. The transient state simulation



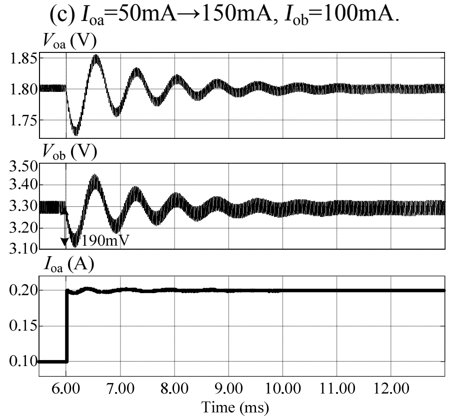
(a) I_{ob}=50mA→150mA, I_{oa}=100mA.



(b) I_{ob}=100mA→200mA, I_{oa}=100mA.

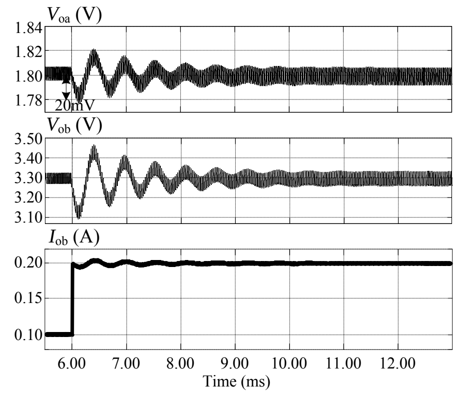


(c) I_{oa}=50mA→150mA, I_{ob}=100mA.

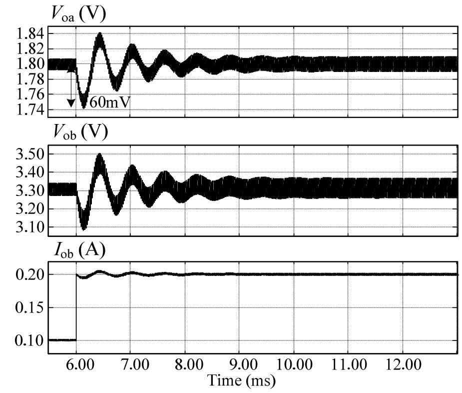


(d) I_{oa}=100mA→200mA, I_{ob}=100mA.

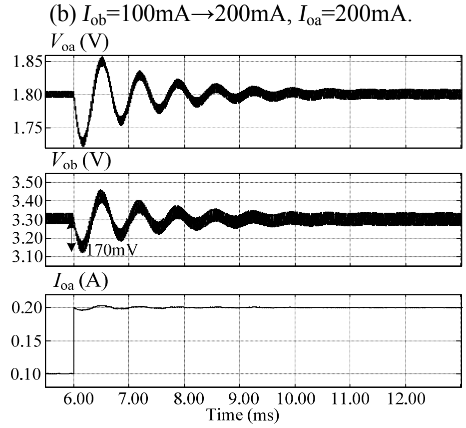
Fig. 10. The transient-state simulation waveforms of cross regulation of output B to output A with different R_b or output A to output B with different R_a.



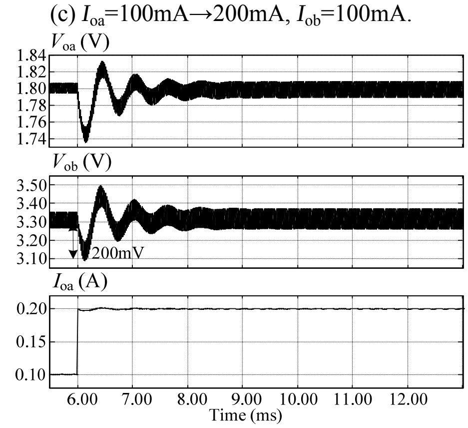
(a) I_{ob}=100mA→200mA, I_{oa}=100mA.



(b) I_{ob}=100mA→200mA, I_{oa}=200mA.



(c) I_{oa}=100mA→200mA, I_{ob}=100mA.



(d) I_{oa}=100mA→200mA, I_{ob}=200mA.

Fig. 11. Transient-state simulation waveforms of cross regulation of output A to output B with different R_b or output B to output A with different R_a.

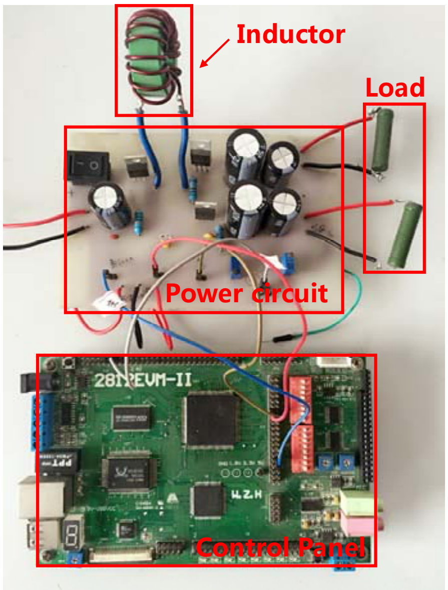


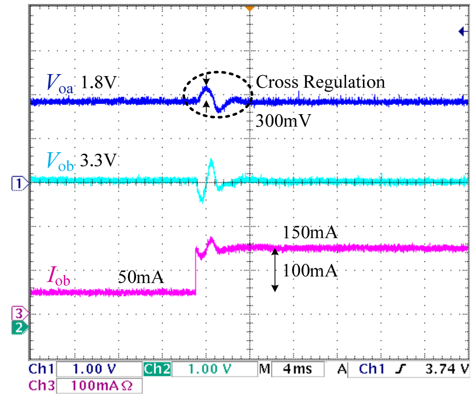
Fig. 12. The prototype for the experiment.

waveform when I_{ob} is equal to 200mA and I_{oa} is stepped changed from 100mA to 200mA is shown in Fig. 11(d). As shown in Fig. 11(d), the cross regulation of output A to output B is 200mV. Thus, it is verified that when output B operates at a lighter load condition, a weaker cross regulation of output A to output B is produced. In addition, when output B operates at a heavier load condition, a greater cross regulation of output A to output B is produced.

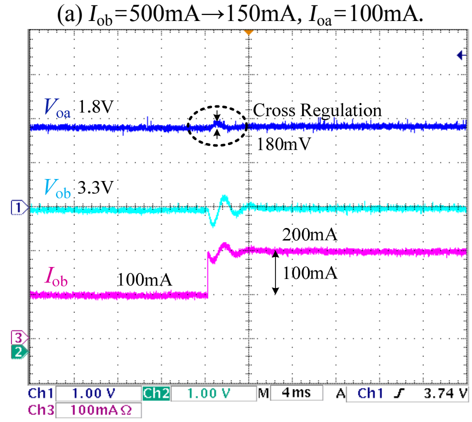
B. Experiment Results

The experimental study of a SIDO CCM Buck converter is employed to verify the analysis and simulation results. The prototype of the experiment is shown in Fig. 12 and the circuit parameters are listed in Table I, which are the same as the ones used in the simulation study. The power stage has been designed based on the circuit diagram shown in Fig. 1. The power switches S_i, S_a and S_b are IRF540N MOSFETs (*n*-channel), and their gates are connected to an IR2110 MOS driver. The diode is a Fast Recovery MUR560. The control signals V_{gsi}, V_{gsa} and V_{gsb} are obtained as the outputs of a microcontroller DSP TMS320F2812, via processing the state variables (V_{oa} and V_{ob}), which are sampled once per period T_s. The microcontroller determines duty ratios considering the constant output voltages within a period. The integral action is implemented digitally. Hence, the PI parameters are loaded into the microcontroller memory. Experimental diagrams corresponding to the simulation results are shown in Fig. 13 and 14, respectively.

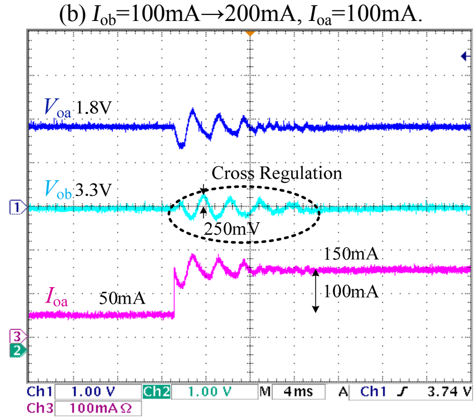
1) Cross Regulation of Output B to Output A with Different R_b or of Output A to Output B with Different R_a.: Fig. 13(a) and Fig. 13(b) show transient waveforms when I_{ob} is step changed from 50mA to 150mA and from 100mA to 200mA.



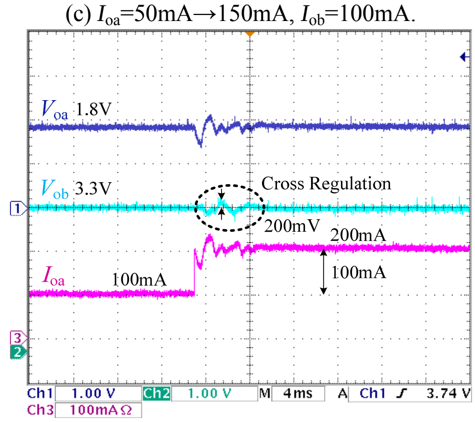
(a) I_{ob}=500mA→150mA, I_{oa}=100mA.



(b) I_{ob}=100mA→200mA, I_{oa}=100mA.



(c) I_{oa}=50mA→150mA, I_{ob}=100mA.

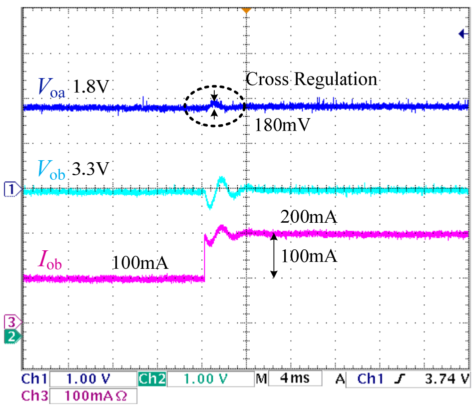


(d) I_{oa}=100mA→200mA, I_{ob}=100mA.

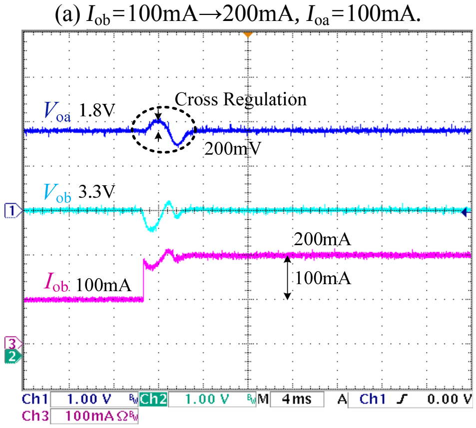
Fig. 13. Transient-state experiment waveforms of cross regulation of output B to output A with different R_b or output A to output B with different R_a.

TABLE II
EXPERIMENT RESULTS

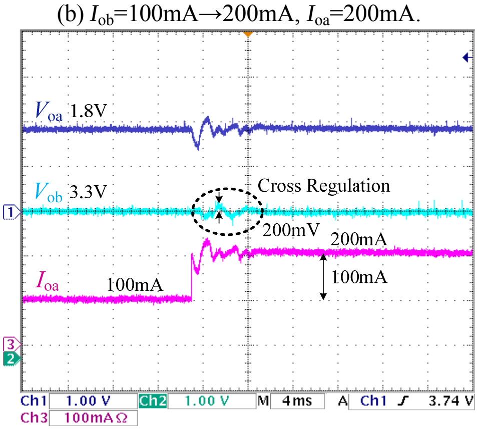
Output current (mA)	Cross Regulation (mV)	
	Output B to Output A	Output A to Output B
I_{ob}=50→150, I_{oa}=100	300	/
I_{ob}=100→200, I_{oa}=100	180	/
I_{oa}=50→150, I_{ob}=100	/	250
I_{oa}=100→200, I_{ob}=100	/	200
I_{ob}=100→200, I_{oa}=100	180	/
I_{ob}=100→200, I_{oa}=200	200	/
I_{oa}=100→200, I_{ob}=100	/	200
I_{oa}=100→200, I_{ob}=200	/	400



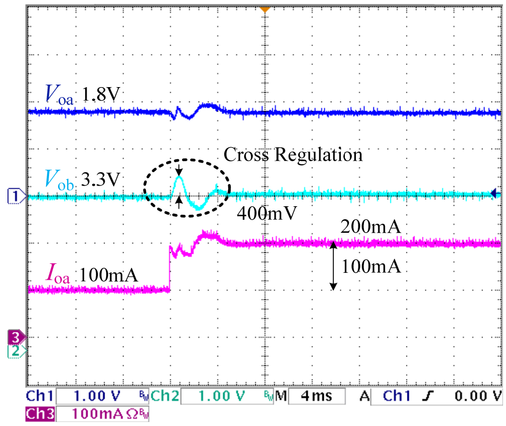
(a) I_{ob}=100mA→200mA, I_{oa}=100mA.



(b) I_{ob}=100mA→200mA, I_{oa}=200mA.



(c) I_{oa}=100mA→200mA, I_{ob}=100mA.



(d) I_{oa}=100mA→200mA, I_{ob}=200mA.

Fig. 14. The transient-state experiment waveforms of cross regulation of output A to output B with different R_b or output B to output A with different R_a.

Fig. 13(c) and Fig. 13(d) show transient waveforms when I_{oa} is step changed from 50mA to 150mA and from 100mA to 200mA. The experiment results of the cross regulation are given in Table II.

2) Cross regulation of output A to output B with different R_b or of output B to output A with different R_a.: Fig. 14(a) and Fig. 14(b) show transient waveforms when I_{ob} is step changed from 100mA to 200mA and when I_{oa} is step changed from 100mA to 200mA. Fig. 14(c) and Fig. 14(d) show transient waveforms when I_{oa} is step changed from 100mA to 200mA and when I_{ob} is step changed from 100mA and 200mA. The experimental results of cross regulation are also given in Table II.

The data in Table II shows that when I_{ob} is step changed from 50mA to 150mA (I_{oa}=100mA) and when I_{oa} is step changed from 50mA to 150mA (I_{ob}=100mA), greater cross regulation of output B to output A and of output A to output B is produced, than when there is a step change from 100mA to 200mA. When output A and output B operate at I_{oa}=100mA (I_{ob}=100mA→200mA) and I_{ob}=100mA (I_{oa}=100mA→200mA), a weaker cross regulation of output B to output A and output A to output B will be produced, when compared with I_{oa}=200mA (I_{ob}=100mA→200mA) and I_{ob}=200mA (I_{oa}=100mA→200mA). Therefore, the experimental results efficiently verify the analysis and simulation results of the SIDO CCM Buck converter. In addition, it is obvious that the transient performance of the converter is good.

VII. CONCLUSIONS

In this paper, a small signal model that is suitable for analyzing the cross regulation of a SIDO CCM Buck converter, is established based on the time averaging equivalent circuit approach. The operation principle of a SIDO CCM Buck converter is introduced and a detailed small signal analysis process is presented. Based on the proposed small signal model, a small signal model of a voltage controlled SIDO CCM Buck converter is established. The law of cross regulation with load changes is studied. It is shown that when the load of output B changes, the disturbance on output A caused by cross regulation is heavier since output A operates at a heavier load condition and output B operates at a lighter load condition. To reduce the cross regulation caused by various load changes in output B, a Buck converter should be designed with a lighter load in output A and a heavier load in output B. When the load of output A changes, the opposite conclusion is reached. Furthermore, to reduce cross regulation based on the small signal model, a current feedback control strategy is developed. The frequency domain analysis shows that the designed feedback controller can significantly reduce cross regulation. Simulation and experimental results obtained from a SIDO Buck converter verified that the law of cross regulation exist in the converter.

ACKNOWLEDGMENT

This work is supported by the 2016 Scientific research project of Sichuan Provincial Department of Education and Fundamental Research Funds for the Central Universities (no. 2015NZYQN15).

REFERENCES

[1] D. Kwon and G. A. Rincn-Mora, "Single-inductor multiple-output switching DC-DC converters," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, Vol. 56, No. 8, pp. 614-618, Aug. 2009.

[2] H. P. Le, C. S. Chae, K. C. Lee, S. W. Wang, and G. H. Cho, "A single-inductor switching DC-DC converter with *five* output and ordered power-distributive control," *IEEE J. Solid-State Circuits*, Vol. 42, No. 12, pp. 2706-2714, Dec. 2007.

[3] D. Trevisan, P. Mattavelli, and P. Tenti, "Digital control of single-*inductor* multiple-output step-down DC-DC converters in CCM," *IEEE Trans. Ind. Electron.*, Vol. 55, No. 9, pp. 3476-3483, Sep. 2008.

[4] K. Y. Lin, C. S. Huang, D. Chen, and K. H. Liu, "Modeling and design of feedback loops for a voltage-mode single-inductor dual-output buck converter," in *IEEE Power Electronics Specialists Conference (PESC)*, pp. 3389-3395, Jun. 2008.

[5] P. Patra, A. Patra, and N. Misra, "A single-inductor multiple-output switcher with simultaneous buck, boost, and inverted outputs," *IEEE Trans. Power Electron.*, Vol. 27, No. 4, pp. 1936-1951, Apr. 2012.

[6] D. S. Ma, W. H. Ki, C. Y. Tsui, and P. K. T. Mok, "Single inductor multiple output switching converters with time multiplexing control in discontinuous conduction mode," *IEEE J. Solid-State Circuits*, Vol. 38, No. 1, pp. 89-100, Jan. 2003.

[7] X. S. Liu, J. P. Xu, J. P. Wang, and Q. B. Mu, "A single-inductor dual-output Buck converter with pulse-train control," in *the 2nd International Symposium on Power Electronics for Distributed Generation Systems (PEDG)*, pp. 449-452, Jun. 2010.

[8] D. S. Ma, W. H. Ki, and C. Y. Tsui, "A pseude-CCM/DCM SIMO switching converter with freewheel switching," *IEEE J. Solid-State Circuits*, Vol. 38, No. 6, pp. 1007-1014, Jun. 2003.

[9] D. S. Ma and W. H. Ki, "Fast-transient PCCM switching converter with freewheel switching control," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, Vol. 54, No. 9, pp. 825-829, Sep. 2007.

[10] Y. Zhang, R. Bondade, D. S. Ma, and S. Abedinpour, "An integrated SIDO Boost power converter with adaptive freewheel switching technique," in *IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 3516-3522, Sep. 2010.

[11] W. W. Xu, Y. Li, Z. L. Hong, and D. Killat, "A dual-mode single-inductor dual-output switching converter with small ripple," *IEEE Trans. Power Electron.*, Vol. 25, No. 3, pp. 614-623, Mar. 2010.

[12] X. C. Jing, P. K. T. Mok, and M. C. Lee, "A wide load-range constant-charge-auto-hopping control single inductor dual output Boost regulator with minimized cross-regulation," *IEEE J. Solid-State Circuits*, Vol. 46, No. 10, pp. 2530-2362, Oct. 2011.

[13] Y. H. Lee, T. C. Huang, Y. Y. Yang, W. S. Chou, K. H. Chen, C. C. Huang, and Y. H. Lin, "Minimized transient and steady-state cross regulation in 55nm CM0S single-inductor dual-output (SIDO) step-down DC-DC converter," *IEEE J. Solid-State Circuits*, Vol. 46, No. 11, pp. 2488-2499, Nov. 2011.

[14] M. H. Huang and K. H. Chen, "Single-inductor multi-output (SIMO) DC-DC converters with high light-load efficiency and minimized cross-regulation for portable devices," *IEEE J. Solid-State Circuits*, Vol. 44, No. 4, pp. 1099-1111, Apr. 2009.

[15] Z. H. Shen, X. G. Chang, W. W. Wang, and H. Ming, "Predictive digital current control of single-inductor multiple-output converters in CCM with low cross regulation," *IEEE Trans. Power Electron.*, Vol. 27, No. 4, pp. 1917-1925, Apr. 2012.

[16] P. Patra, J. Ghosh, and A. Patra, "Control scheme for reduced cross regulation in single-inductor multiple-output DC-DC converters," *IEEE Trans. Ind. Electron.*, Vol. 60, No. 11, pp. 5095-5104, Nov. 2013.

[17] J. D. Dasika, B. Bahrani, M. Saeedifard, A. Karimi, and A. Rufer, "Multivariable control of single-inductor dual-output Buck converters," *IEEE Trans. Power Electron.*, Vol. 29, No. 4, pp. 2061-2070, Apr. 2014.

[18] R. W. Erickson, *Fundamentals of power electronics*, New York Chapman & Hall, 1997.

[19] C. S. Huang, D. Chen, C. J. Chen, and K. H. Liu, "Mix-voltage conversion for single-inductor dual-output buck converters," *IEEE Trans. Power Electron.*, Vol. 25, No. 8, pp. 2106-2114, Aug. 2010.

Yao Wang was born in Sichuan, China. She received her B.S. degree in Electrical Engineering and Automation from the Sichuan Normal University, Chengdu, China, in 2011; and her M.S. degree in Electrical Engineering from the Southwest Jiaotong University Chengdu, China, in 2014. Since 2014, she has been an Assistant Experimentalist in the School of Electrical Engineering and Information Engineering of the Southwest University for Nationalities, Chengdu, China. Her current research interests include the modeling, control and dynamic behavior analysis of power electronic systems.

Jianping Xu was born in Zunyi, China. He received his B.S. and Ph.D. degrees in Electronic Engineering from the University of Electronics Science and Technology of China, Chengdu, China, in 1984 and 1989, respectively. Since 1989, he has been with the School of Electrical Engineering, Southwest Jiaotong University, Chengdu, China, where he has been a Professor, since 1995. From November 1991 to February 1993, he was a Visiting Research Fellow in the Department of Electrical Engineering, University of the German Federal Armed Forces, Munich, Germany. From February 1993 to July 1994, he was a Visiting Scholar in the Department of Electrical Engineering and Computer Science, University of Illinois, Chicago, IL, USA. His current research interests include the modeling, analysis, and control of power electronic systems.

Guohua Zhou received his B.S. degree in Electronic and Information Engineering, and his M.S. and Ph.D. degrees in Electrical Engineering from the Southwest Jiaotong University, Chengdu, China, in 2005, 2008 and 2011, respectively, where he is presently working as a Professor in the School of Electrical Engineering. From March 2010 to September 2010, he was a Research Assistant in the Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Kowloon, Hong Kong, China. From October 2010 to March 2011, he was a Visiting Scholar (joint Ph.D. student) with the Center for Power Electronics Systems, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA. His major fields of study are Power Electronics and Electric Drive. His current research interests include the analog control, digital control and modulation methods of switching power converters, modeling and analysis of the dynamic behavior of switching dc-dc converters, and renewable energy applications for power electronics.