

# Improving the Solution Range in Selective Harmonic Mitigation Pulse Width Modulation Technique for Cascaded Multilevel Converters

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## Abstract

This paper proposes an improved low frequency Selective Harmonic Mitigation-PWM (SHM-PWM) technique. The proposed method mitigates the low order harmonics of the output voltage up to the 50<sup>th</sup> harmonic well and satisfies the grid codes EN 50160 and CIGRE-WG 36-05. Using a modified criterion for the switching angles, the range of the modulation index for non-linear SHM equations is improved, without increasing the switching frequency of the CHB converter. Due to the low switching frequency of the CHB converter, mitigating the harmonics of the converter up to the 50<sup>th</sup> order and finding a wider modulation index range, the size and cost of the passive filters can be significantly reduced with the proposed technique. Therefore, the proposed technique is more efficient than the conventional SHM-PWM. To verify the effectiveness of the proposed method, a 7-level Cascaded H-bridge (CHB) converter is utilized for the study. Simulation and experimental results confirm the validity of the above claims.

**Key words:** Cascaded H-bridge inverter, Multilevel converter, Particle swarm optimization, Selective harmonic mitigation

## I. INTRODUCTION

Multilevel converters have gained a lot of attention in recent years due to their salient features such as lower stress across the semiconductors, lower common-mode voltage generation, lower harmonics in the output waveform, and lower EMI generation [1], [2]. In all of the multilevel structures, a stepwise voltage waveform is synthesized to reduce the total harmonic distortion (THD) and harmonic content. These converters are categorized as diode clamp converter, flying capacitor converter and cascaded H-bridge (CHB) converter. Among these topologies, the CHB has the highest modularity and it can be easily scaled to different voltage and power levels. Therefore, it is selected for further study in this paper.

In the applications such as Flexible AC Transmission Systems (FACTS) [3], High Voltage DC lines (HVDC) [4], and electrical drives [5], it is important to achieve high

efficiencies while the THD is minimum. This goal can be achieved through employing low frequency modulation techniques for the CHB converters. According to [6]-[18], selective harmonic elimination (SHE), selective harmonic mitigation (SHM) and selective harmonic current mitigation PWM [31], [32] are the most prominent low frequency modulation techniques. The implementation of these methods in CHB converter has met with the following challenges:

- 1) Due to a large number of non-linear equations, solving and finding answers is not simple.
- 2) To limit the switching loss, a minimum number of switching transitions should be utilized to satisfy grid codes.

In the SHM approach, instead of complete elimination of low order harmonics like the SHE method, the non-linear equations are solved to keep the low order harmonics smaller than the specified limits in grid codes. Hence, as an advantage, the possibility of finding solutions for a wide range of modulation indices is higher. It is also worth mentioning that in conventional SHM, each cell has one switching transition in a quarter-cycle. However, in selective harmonic mitigation-PWM (SHM-PWM) the number of switching transitions can be more than one. This feature helps to mitigate more number of harmonics without increasing the number of

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H-bridge cells in the CHB converter. Thus, the SHM-PWM is selected for further study and adoption in this article.

The SHM-PWM technique was first proposed in [12] for a CHB inverter. The switching frequency of each switch was limited to 750 Hz and the low order harmonics (lower than 50) were mitigated in a three-phase inverter. In [13], a closed loop implementation of the proposed idea in [14] was carried out and the results were satisfactory. The voltage of the capacitors can be variable to increase the degrees of freedom [9], [15]-[20]. However, in this method, the inverter should be fed by variable DC sources which results in a lot of implementation difficulties.

A SHM-PWM approach based on equal DC-link voltages in a CHB inverter was introduced in [21]. It uses nine switching angles in each quarter-cycle. Therefore, the switching frequency of each power switch is limited to 150 Hz. However, this method can only eliminate non-triplen harmonics up to the 40<sup>th</sup> term.

Conventionally, to obtain SHE-PWM and SHM-PWM equations for the CHB inverters, the switching angles of H-bridge cells are usually arranged sequentially. However, in [11], [22], a new idea for the SHE-PWM technique has been presented, which tries to eliminate the above criterion. The main goal of this idea is to increase the search space for the mathematical solver and to have a wider range of available solutions.

In [23], a combination of the SHM and SHE modulation techniques was proposed, where a technique was used to produce appropriate waveforms for a four-leg three-level NPC inverter. In this study, by using SHM technique, the non-triplen harmonics in the phase legs are mitigated. In addition, the fourth leg is controlled by SHE to eliminate important low-order triplen harmonics. In [24], an inverter scheme of a low frequency modulation index was proposed for high power applications. Moreover, the SHE and SHM techniques have been used in wide variety of applications [25].

In this paper, the idea of non-sequential switching angles is extended to the SHM-PWM technique to improve the solution range. A lot of effort has been devoted to mitigate non-triplen low order harmonics up to the 50<sup>th</sup> term by just nine switching angles in a quarter-cycle of the fundamental period. In other words, the switching frequency of the switches is limited to 150 Hz in a 7-level CHB inverter. Furthermore, the grid codes EN 50160 and CIGRE-WG 36-05 [21], [26], [27], are considered in the development of the proposed SHM-PWM method. The validity of the proposed method is verified by simulations and experiments on a 7-level CHB inverter.

## II. CASCADED H-BRIDGE STRUCTURE

Among the various multilevel converters, the CHB converter has a modular structure and requires the minimum number of components to synthesize the same number of voltage levels. As is shown in Fig. 1, the CHB inverter is made of N series

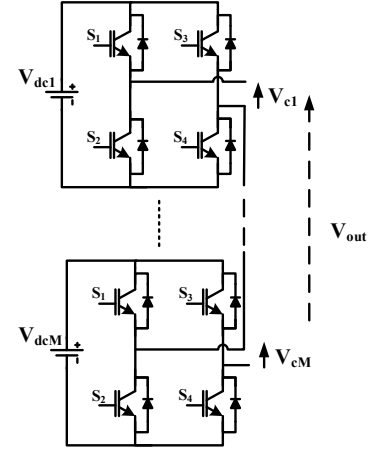


Fig. 1. Structure of a CHB inverter.

TABLE I  
MAIN SWITCHING STATES IN AN H-BRIDGE CELL

| $V_{ac}$  | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|-----------|-------|-------|-------|-------|
| $+V_{dc}$ | 1     | 0     | 0     | 1     |
| 0         | 1     | 0     | 1     | 0     |
| 0         | 0     | 1     | 0     | 1     |
| $-V_{dc}$ | 0     | 1     | 1     | 0     |

connected H-bridge cells. The CHB inverter tries to synthesize the desired AC output voltage ( $V_{out}$ ) from the distinct DC voltages, i.e.,  $V_{dc1}$ ,  $V_{dc2}$ , ...,  $V_{dcM}$ . In the symmetric CHB structures, the voltages of all DC links are considered to be equal, and this assumption is used in this article. Hereafter, the DC link voltages are assumed to be  $V_{dc}$ .

Each H-bridge cell in Fig. 1 can generate three different voltage levels, i.e.,  $V_{dc}$ , 0, and  $-V_{dc}$  at its AC terminals. Table I shows different switching states which can be used to synthesize a voltage level at the cell AC terminal. It is evident that the total number of voltage levels that can be synthesized at the inverter phase AC voltage is  $2M+1$ , where  $M$  is the number of H-Bridge cells.

## III. SELECTIVE HARMONIC MITIGATION MODULATION (SHM-PWM) FOR CHB INVERTERS

### A. Basic Equations of the SHM-PWM Technique

In the SHM-PWM method, first a cycle of a predefined voltage waveform is considered. Then using the Fourier series analysis formula, the Fourier series coefficients of the voltage waveform are calculated. Next, the amplitude for each of the voltage harmonics is determined and they are used in the non-equality equations of SHM-PWM, except the first harmonic which is used to control the amplitude of the output voltage. In these equations, the upper limits are determined according to selected grid codes, which should be satisfied.

The general equation of the SHM-PWM method is shown in the following:

$$V(\alpha) = \sum_{n=1}^{\infty} (a_n \cos(n\alpha) + b_n \sin(n\alpha)) \quad (1)$$

where  $\omega$  is the output frequency in radians. In addition,  $a_n$  and  $b_n$  are coefficients of the Fourier series. In the predefined waveform in Fig. 2, the  $a_n$  coefficients are eliminated in the output harmonic spectra, because of the quarter-wave symmetry. Hence, the Fourier series expansion of the predefined waveform is simplified as:

$$V(\alpha) = \sum_{n=1}^{\infty} b_n \sin(n\alpha) \quad (2)$$

where the  $b_n$  coefficients are:

$$b_n = \begin{cases} 0 & n = \text{even} \\ \frac{4V_{dc}}{n\pi} \sum_{\substack{i=1, \dots, m \\ j=1, \dots, n_i}} K_{ij} \cos n\theta_{ij} & n = \text{odd} \end{cases} \quad (3)$$

where  $K_{ij}$  is +1 if the  $i^{\text{th}}$  transition edge in Fig. 2 is rising, and it is -1 if the transition edge is falling. The main purpose of the SHM-PWM method is to control the amplitude of the fundamental harmonic and to mitigate the selected harmonics from the output voltage. The first coefficient in (3), i.e.,  $b_1$  controls the amplitude of the fundamental harmonic (or the modulation index  $m_a$ ), and it is determined by:

$$b_1 = \frac{4V_{dc}}{\pi} (\cos \theta_{11} - \cos \theta_{12} + \dots + \quad (4)$$

$$\cos \theta_{1n_1} + \dots + \cos \theta_{m1} + \dots + \cos \theta_{mn_m})$$

$$m_a = (\cos \theta_{11} - \cos \theta_{12} + \dots + \cos \theta_{1n_1} + \quad (5)$$

$$\cos \theta_{21} \dots + \cos \theta_{m1} + \dots + \cos \theta_{mn_m})$$

where  $m_a$  represents the modulation index and it can vary between 0 and  $m$  for a  $2m+1$  level CHB converter. In this paper, a 7-level waveform is considered and shown in Fig. 3. The SHM-PWM equations are defined to satisfy the grid codes EN 50160 and CIGRE-WG 36-05 [15], [30], [31]. The limits of these codes are shown in Table II. As can be seen from Table II, the THD value must be restricted to 8%.

According to (2-5), the SHM-PWM equations for the predefined waveform in Fig. 3 are derived as:

$$\begin{cases} m_a = \cos \theta_{11} - \cos \theta_{12} + \cos \theta_{13} + \cos \theta_{21} \\ \quad - \cos \theta_{22} + \cos \theta_{23} + \cos \theta_{31} - \cos \theta_{32} + \cos \theta_{33} \\ H_1 = \frac{4V_{dc}}{\pi} (\cos \theta_{11} - \cos \theta_{12} + \cos \theta_{13} + \cos \theta_{21} \\ \quad - \cos \theta_{22} + \cos \theta_{23} + \cos \theta_{31} - \cos \theta_{32} + \cos \theta_{33}) \\ \left[ \frac{4V_{dc}}{m\pi} \begin{pmatrix} \cos(m\theta_{11}) - \cos(m\theta_{12}) + \cos(m\theta_{13}) \\ + \cos(m\theta_{21}) - \cos(m\theta_{22}) + \cos(m\theta_{23}) \\ + \cos(m\theta_{31}) - \cos(m\theta_{32}) + \cos(m\theta_{33}) \end{pmatrix} \right] \leq |H_1| L_m \\ m=5,7,11,13,17,19,23,25,29,31,35,37,41,43,47,49 \end{cases} \quad (6)$$

where  $H_1$  is the amplitude of the fundamental harmonic and  $L_m$  determines the upper limit of the  $m^{\text{th}}$  harmonic according to the standards.

### B. Proposed SHM-PWM Algorithm

Generally, in a CHB converter, each level of the output

TABLE II  
GRID CODES: EN 50160 AND CIGRE WG 36-05

| Harmonic limits         |                    |                   |                    |
|-------------------------|--------------------|-------------------|--------------------|
| Non-triplen harmonics   |                    | Triplen harmonics |                    |
| Harmonic order, n       | Voltage limits, Li | Harmonic order, n | Voltage limits, Li |
| 5                       | 6%                 | 3                 | 5%                 |
| 7                       | 5%                 | 9                 | 1.5%               |
| 11                      | 3.5%               | 15                | 0.5%               |
| 13                      | 3%                 | 21                | 0.5%               |
| 17                      | 2%                 | >21               | 0.2%               |
| 19                      | 1.5%               |                   |                    |
| 23                      | 1.5%               |                   |                    |
| 25                      | 1.5%               |                   |                    |
| >25                     | 0.2+32.5/n         |                   |                    |
| THD (40 <sup>th</sup> ) | 8%                 |                   |                    |

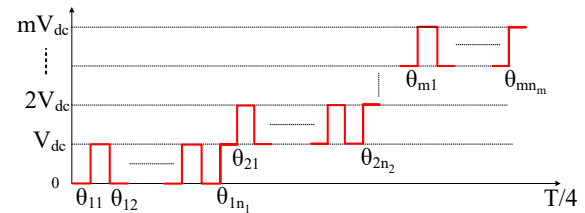


Fig. 2. Predefined voltage waveform of the SHM-PWM.

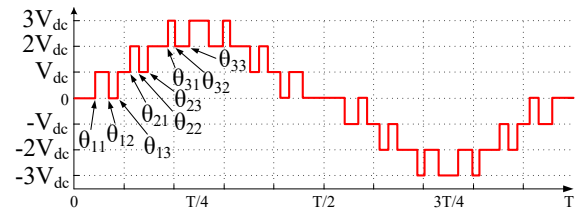


Fig. 3. A 7-level predefined voltage waveform in SHM-PWM.

waveform is generated by a specific cell. For example, in a  $2m+1$  level converter, like the one in Fig. 1 and for the predefined waveform in Fig. 2, the first, second and  $m^{\text{th}}$  H-bridge cells should contribute to the modulation in the phase intervals of  $(\theta_{11} - \theta_{1n_1})$ ,  $(\theta_{21} - \theta_{2n_2})$  and  $(\theta_{m1} - \theta_{mn_m})$ , respectively. This contribution can be shown by:

$$0 < \overbrace{\theta_{11} < \dots < \theta_{1n_1}}^{\text{Cell 1}} < \dots < \overbrace{\theta_{m1} < \dots < \theta_{mn_m}}^{\text{Cell m}} < \frac{\pi}{2} \quad (7)$$

According to the above rule, the 1<sup>st</sup>, 2<sup>nd</sup>, ...,  $m-1^{\text{th}}$  cells are turned on more than the last cell. Therefore, their power losses are unequal. As a result, this can reduce the reliability of the converter due to different power losses. Thus, the ON-time of the switches is a key factor in the design of SHM-PWM technique in industrial applications.

Furthermore, solving the non-linear equations in (6) and finding solutions over a wide range of modulation indices are not easy. However, in this paper, another rule is used for the sequence of switching, which helps to get more degrees of freedom for solving the non-linear equations in the

SHM-PWM method [23]. In addition, it also results in a better distribution of power loss. In the proposed rule, for a specific modulation index, three arbitrary switching angles are devoted to each of the H-bridge cells. As an example, the following arrangement may be applied for a specific modulation index:

$$\begin{aligned} & \text{Cell 1} \\ & 0 < \overbrace{\theta_{11} < \dots < \theta_{1n_1}} < \frac{\pi}{2} \\ & \text{Cell 2} \\ & 0 < \overbrace{\theta_{21} < \dots < \theta_{2n_2}} < \frac{\pi}{2} \\ & \vdots \\ & \text{Cell } m \\ & 0 < \overbrace{\theta_{m1} < \dots < \theta_{mn_m}} < \frac{\pi}{2} \end{aligned} \quad (8)$$

According to (8), there is no constraint for the H-bridge cells to be involved in the modulation sequentially. In other words, each of the H-bridge cells can contribute to the transitions at  $n_i$  ( $i=1, \dots, m$ ) arbitrary switching angles. This feature gives more relaxation to the mathematical solver in finding solutions to non-linear equations.

It is worth mentioning that for a specific modulation index, the command of the switching transitions can be changed between H-bridge cells in a regular way to equally distribute the power loss among them in either the conventional or proposed method in this paper.

### C. Solving SHM-PWM Equations

One of the key challenges associated with SHM-PWM is to solve the non-linear equations with the trigonometric terms. Many different methods have been proposed to solve these equations [28]–[30]. In this paper, particle swarm optimization (PSO) algorithm [21] is used to solve them.

The main difference between these techniques is in the accuracy and speed of the optimization technique in terms of finding solutions. The PSO technique uses the best local particles (solutions) in each iteration to guide the other particles (solutions) in order to find the best global particle. Consequently, in each iteration the objective functions of all the particles are checked to find the local and global best particles. The main equations which are used in the PSO technique are shown in the following equations:

$$v_j^{k+1} = wv_j^k + c_1 r_1 (pbest_j^k - x_j^k) + c_2 r_2 (gbest^k - x_j^k) \quad (9)$$

$$x_j^{k+1} = x_j^k + v_j^{k+1} \quad (10)$$

where  $w$  is a weighted factor which is determined by the iteration number.  $x$  and  $v$  are the position and velocity for each of the particles, respectively. Moreover, in each iteration, these parameters should be updated, while the best position of the local and global swarms are stored in the  $pbest$  and  $gbest$  parameters, respectively. The objective function is defined as follows [19]:

$$OF(\theta_{11}, \theta_{12}, \dots, \theta_{33}) = \begin{cases} H_i \leq L_i |H_i| \lambda, & i = 5, 7, 11, \dots, 49 \\ H_1 = \frac{4V_{dc}}{\pi} m_a \end{cases} \quad (11)$$

where  $H_i$  represents the  $i^{\text{th}}$  objective function to be minimized

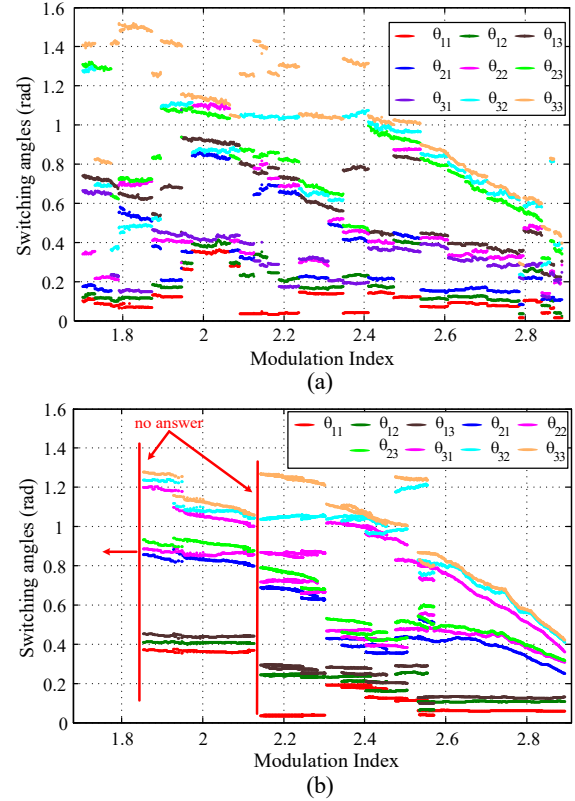


Fig. 4 Obtained solutions. (a) Proposed SHM-PWM. (b) Conventional SHM-PWM.

and  $\lambda$  is a real factor that is lower than unity. If  $\lambda = 1$ , the corresponding harmonic will be same as the defined limits in Table II.

For an interval of  $1.7 < m_a < 2.89$ , the corresponding equations are solved and the results are shown in Fig. 4(a). It can be seen that there are always answers for modulation indices between 1.7 and 2.89. Meanwhile, in the conventional approach, there are no answer in some of the modulation indices (e.g., in  $m_a = 2.13$ ) or in  $m_a < 1.84$ . Fig. 4(b) shows the obtained answer sets by the conventional approach. It is also worth mentioning that it is possible to find different solutions based on the proposed approach for a specific modulation index, which gives more flexibility in the selection of answers from the view point of the THD and power losses.

The obtained solutions, which are shown in Fig. 4, always meet the requirements of the standards for both the conventional and the proposed technique. As long as a solution meets the requirements of the standards for all of the harmonics up to the 50<sup>th</sup> and the THD, it can be considered as a solution. The main objective of this paper is to increase the solution range of the SHM-PWM technique, not to make harmonic magnitudes less than those of the conventional technique. Therefore, for some harmonic orders of the proposed technique, the harmonic magnitudes can be higher than those of the conventional technique. However, because these harmonics meet the requirements of the standards, there is no issue in terms of them being considered as a solution for

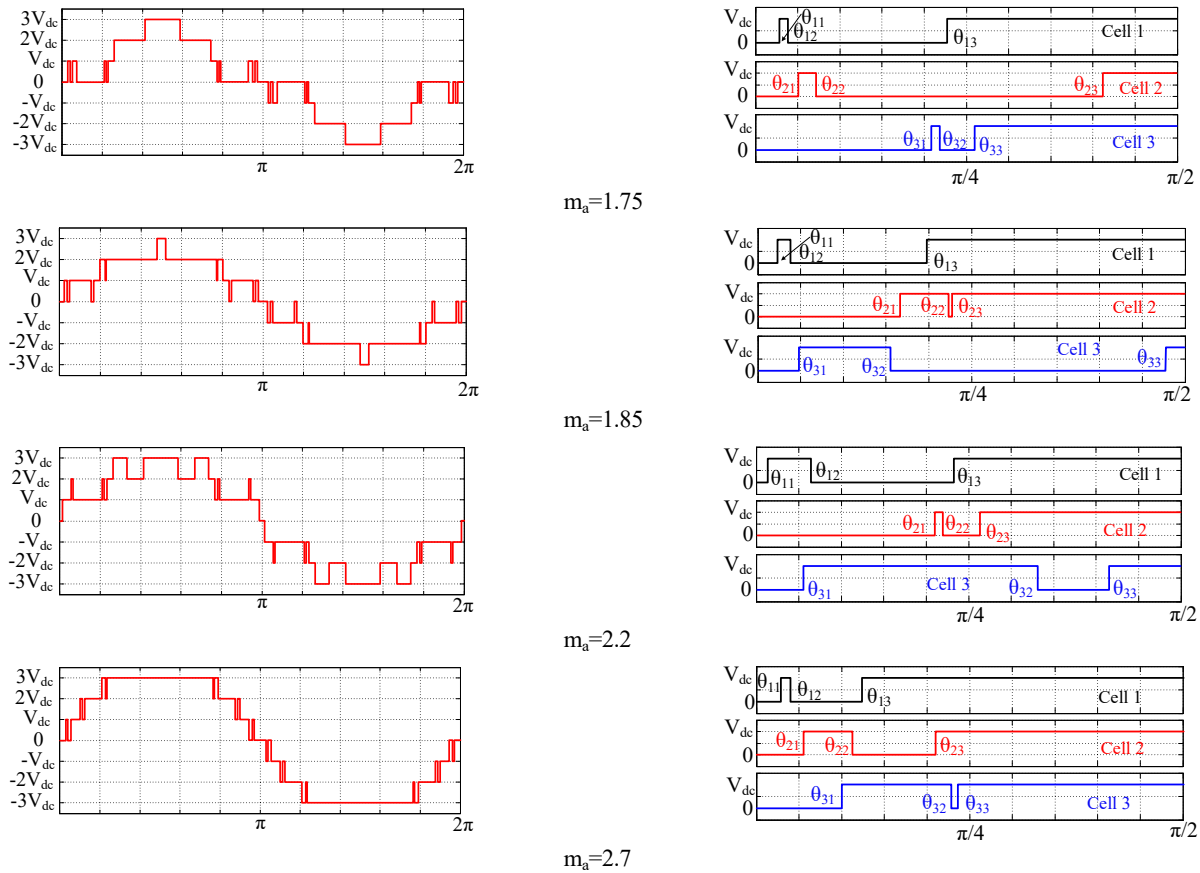


Fig. 5. Output phase voltage (left) and its corresponding voltage for each of the H-bridge cells (right) at different modulation indices.

TABLE III  
SWITCHING ANGLES OF THE PROPOSED METHOD WITH DIFFERENT MODULATION INDICES

| Modulation Index | $\theta_{11}$ | $\theta_{12}$ | $\theta_{13}$ | $\theta_{21}$ | $\theta_{22}$ | $\theta_{23}$ | $\theta_{31}$ | $\theta_{32}$ | $\theta_{33}$ |
|------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1.7              | 0.103366      | 0.121309      | 0.741659      | 0.176030      | 0.342558      | 1.311165      | 0.666244      | 1.280078      | 1.408369      |
| 1.75             | 0.087246      | 0.118919      | 0.714617      | 0.158518      | 0.225411      | 1.293509      | 0.654947      | 0.686776      | 0.816049      |
| 1.85             | 0.070206      | 0.117099      | 0.620986      | 0.52276       | 0.701065      | 0.713281      | 0.148911      | 0.486899      | 1.502086      |
| 1.9              | 0.121940      | 0.171233      | 0.680752      | 0.206532      | 0.409360      | 1.088634      | 0.451172      | 1.101606      | 1.427351      |
| 2.2              | 0.039570      | 0.200946      | 0.731467      | 0.660646      | 0.689968      | 0.827511      | 0.173996      | 1.03996       | 1.30489       |
| 2.5              | 0.122271      | 0.407316      | 0.841304      | 0.440196      | 0.879644      | 0.933045      | 0.368412      | 0.974253      | 1.017620      |
| 2.7              | 0.089698      | 0.125310      | 0.389775      | 0.173063      | 0.353182      | 0.659427      | 0.314969      | 0.716731      | 0.741867      |
| 2.745            | 0.076250      | 0.105631      | 0.351695      | 0.146971      | 0.320778      | 0.603458      | 0.275462      | 0.648835      | 0.684992      |
| 2.89             | 0.015         | 0.025432      | 0.303608      | 0.109799      | 0.286864      | 0.341842      | 0.248657      | 0.369296      | 0.397888      |

the SHM-PWM.

Extending the solution range of the SHM-PWM technique does not relate to the optimization technique used to solve (11). The only parameter that can extend the solution range is to increase the constraints of the objective function. Therefore, in this paper, by extending the switching angle constraints of the SHM-PWM, the range of the obtained solutions are extended as shown in Fig. 4 (a).

In Fig. 5, the output phase voltage (left) and corresponding contribution of the H-bridge cells (right) are shown for some specific modulation indices. As shown, the contribution of different H-bridge cells varies as a function of the modulation index. The corresponding switching angles are shown in Table III.

The proposed method helps to achieve a better distribution of the power loss among the H-bridge cells when compared to the conventional approach. To verify this behavior, the ON times of different H-bridge cells in a quarter period is measured in radian and shown in Fig. 6. In addition, the ON time as a function of the modulation index for both the conventional and the proposed approaches are also shown. As can be seen, the variance of the proposed method is considerably smaller than that of the conventional SHE-PWM, especially for high modulation indices. Therefore, the power losses of the proposed technique are distributed more evenly between the cells of the converter. As a result, it can improve the reliability of the proposed technique than the conventional technique.

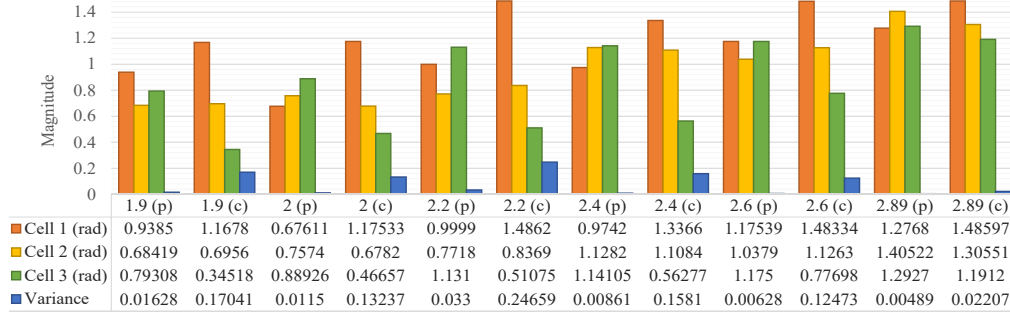


Fig. 6. ON times of different cells as a function of the modulation index in both the proposed approach and the conventional SHE-PWM (p=proposed, c=conventional).

TABLE IV  
SIMULINK AND EXPERIMENTAL PARAMETERS

| Parameter                                   | Symbol    | Value   |
|---|-----------|---------|
| Number of H-Bridge modules                  | N         | 3       |
| Nominal DC link voltage of the CHB inverter | $V_{DC}$  | 30 V    |
| AC side voltage frequency                   | f         | 50 Hz   |
| Switching frequency of inverter             | $f_{s-i}$ | 150 Hz  |
| Capacitance                                 | C         | 4 mF    |
| Power MOSFET                                | S         | IRF540N |

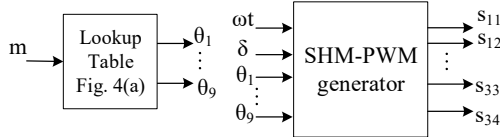


Fig. 7. Associated SHM-PWM generator.

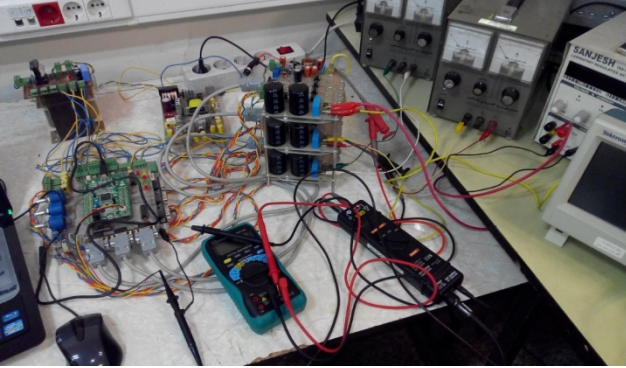


Fig. 8. Hardware prototype for the experimental investigation.

#### IV. SIMULATION AND EXPERIMENTAL RESULTS

In this paper, to verify the effectiveness of the proposed SHM-PWM method, several simulations and experiments have been carried out on a 7-level CHB inverter. The MATLAB Simulink environment has been used to simulate the 7-level converter. The controlling system is shown in Fig. 7. Moreover, in the practical implementation, a CORTEX M4 ARM processor is used as a controller to implement the SHM-PWM algorithm. The parameters of the simulation and hardware prototype are shown in Table. IV. In addition, the hardware prototype in the experiments is shown in Fig. 8.

To show the validity of the proposed method, some simulations are carried out based on the switching angles in Table III for different modulation indices. The results of the simulations are shown in Table V. The MATLAB FFT toolbox is used to calculate the harmonics based on the data that is extracted from oscilloscope. The corresponding harmonic analysis of the experimental phase voltages are also given in Table VI. To compare experimental results of the proposed and conventional methods, the same tests are carried out and reported for both of them. As shown in Table VI, the proposed SHM-PWM method can mitigate the low order harmonics of the output voltage up to the 50<sup>th</sup> order. Furthermore, the triplen harmonics are automatically be eliminated from the harmonic spectra when a 3-phase inverter is used. As can be seen, the proposed method successfully fulfills the grid code requirements, while the switching frequency is 150 Hz. Fig. 9 demonstrates the harmonic spectra of the output waveform at  $m_a=2.7$ . Moreover, waveforms of the practical implementation for modulation indices of  $m_a=1.9$ , 2.4 and 2.745 are shown in Fig. 10. The harmonic spectra and THD of the waveforms in Fig. 10 can be found in Table VI.

#### V. DISCUSSION

The proposed mitigation technique meets the voltage harmonic distortion limits of power quality standards instead of completely eliminating the low order harmonics of the CHB voltage. Therefore, the number of harmonics which can be mitigated in the SHM-PWM is higher than  $2k-1$ , where  $k$  is the number of switching transitions of a single-phase CHB in each quarter of a period. However, to find the exact number of harmonics that can be mitigated with  $k$  number of switching transitions, the optimization technique should be solved and checked with trial and error. As demonstrated in this paper, with 9 switching transitions in each quarter of a period, the conventional SHM-PWM technique has a very limited solution range ( $1.84 < m_a < 2.89$  with some unsolved points). To solve this issue in the proposed technique, the constraints of the switching angles of the converter are improved. As a result, the solution range is increased to ( $1.7 < m_a < 2.89$ ).



TABLE V

THD AND HARMONIC COMPONENTS OF THE SIMULATION RESULTS FOR THE PROPOSED AND CONVENTIONAL SHM-PWM METHODS  
(1. Maximum THD case; 2. Minimum THD case)

| Harmonic order      | Grid code limits,% | 1.7 <sup>1</sup> | 1.75  | 1.85  | 1.9   |       | 2.2   |       | 2.5   |       | 2.7   |      | 2.745 <sup>2</sup> |      | 2.89 |      |      |
|---------------------|--------------------|------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|--------------------|------|------|------|------|
|                     |                    | p                | p     | p     | c     | p     | c     | p     | c     | p     | c     | p    | c                  | p    | c    |      |      |
|                     |                    | p                | p     | p     | c     | p     | c     | p     | c     | p     | c     | p    | c                  | p    | c    |      |      |
| 5                   | 6                  | 3.57             | 2.27  | 1.06  | 3.4   | 1.55  | 2.10  | 1.31  | 0.03  | 3.17  | 0.86  | 2.72 | 0.48               | 0.82 | 3.23 | 5.7  | 5.84 |
| 7                   | 5                  | 4.93             | 4.66  | 0.48  | 5     | 2.41  | 4.42  | 4.65  | 4.68  | 2.73  | 2.92  | 0.72 | 2.99               | 0.43 | 3.4  | 1.8  | 1.54 |
| 11                  | 3.5                | 2.54             | 1.25  | 3.27  | 2.24  | 1.76  | 2.94  | 1.34  | 2.18  | 0.89  | 1.72  | 1    | 2.13               | 0.02 | 3.47 | 3.33 | 2.79 |
| 13                  | 3                  | 2.93             | 2.83  | 2.51  | 1.56  | 1.31  | 0.77  | 2.79  | 2.69  | 1.75  | 2.87  | 2.64 | 2.39               | 2.3  | 2.58 | 1.56 | 0.91 |
| 17                  | 2                  | 0.4              | 0.26  | 0.35  | 2     | 0.32  | 0.23  | 0.04  | 0.95  | 0.77  | 0.99  | 1    | 1.32               | 0.34 | 0.47 | 0.71 | 1.56 |
| 19                  | 1.5                | 1.22             | 1.45  | 1.19  | 0.11  | 1.03  | 0.94  | 0.56  | 1.24  | 1.23  | 1.38  | 1.22 | 1.26               | 0.81 | 0.91 | 0.57 | 1.48 |
| 23                  | 1.5                | 0.67             | 1.2   | 1.05  | 1.5   | 1.4   | 0.8   | 1.04  | 0.02  | 1.23  | 1.27  | 0.95 | 1.38               | 0.2  | 0.23 | 0.64 | 0.3  |
| 25                  | 1.5                | 1.32             | 1.3   | 0.43  | 0.02  | 0.83  | 1.04  | 0.88  | 0.14  | 1.29  | 0.89  | 0.73 | 1.15               | 0.33 | 1.47 | 0.96 | 0.03 |
| 29                  | 1.32               | 0.63             | 0.68  | 0.92  | 0.57  | 0.39  | 0.27  | 1.23  | 0.19  | 1.16  | 1.31  | 1    | 0.83               | 0.18 | 0.62 | 0.81 | 0.21 |
| 31                  | 1.25               | 0.87             | 0.87  | 0.35  | 1.24  | 1.11  | 0.46  | 1.14  | 0.11  | 0.07  | 0.55  | 0.97 | 0.23               | 0.27 | 0.94 | 0.69 | 0.44 |
| 35                  | 1.13               | 0.5              | 0.72  | 0.18  | 1.12  | 0.2   | 0.69  | 1.05  | 0.04  | 0.89  | 0.83  | 1.03 | 0.03               | 0.67 | 1.13 | 0.72 | 0.34 |
| 37                  | 1.08               | 0.25             | 0.33  | 0.1   | 1.07  | 0.53  | 0.32  | 0.28  | 1.08  | 0.95  | 0.36  | 0.58 | 0.1                | 0.54 | 0.22 | 0.68 | 0.03 |
| 41                  | 0.99               | 0.32             | 0.24  | 0.31  | 0.99  | 0.68  | 0.83  | 0.3   | 0.98  | 0.6   | 0.9   | 0.35 | 0.98               | 0.57 | 0.72 | 0.14 | 0.83 |
| 43                  | 0.96               | 0.44             | 0.59  | 0.57  | 0.37  | 0.85  | 0.5   | 0.81  | 0.74  | 0.06  | 0.11  | 0.85 | 0.42               | 0.84 | 0.13 | 0.2  | 0.95 |
| 47                  | 0.89               | 0.17             | 0.76  | 0.63  | 0.6   | 0.66  | 0.46  | 0.07  | 0.89  | 0.64  | 0.88  | 0.16 | 0.82               | 0.14 | 0.26 | 0.52 | 0.26 |
| 49                  | 0.86               | 0.15             | 0.69  | 0.34  | 0.85  | 0.53  | 0.72  | 0.3   | 0.82  | 0.25  | 0.82  | 0.16 | 0.59               | 0.67 | 0.8  | 0.55 | 0.31 |
| THD <sup>40th</sup> | 8                  | 7.58             | 6.61  | 4.72  | 7.45  | 4.36  | 5.99  | 6.26  | 6.21  | 5.48  | 5.42  | 4.83 | 5.26               | 2.87 | 6.92 | 7.32 | 7.13 |
| THD <sup>50th</sup> |                    | 7.6              | 6.71  | 4.81  | 7.53  | 4.52  | 6.07  | 6.32  | 6.37  | 5.53  | 5.56  | 4.91 | 5.38               | 3.06 | 6.92 | 7.36 | 7.2  |
| THD                 |                    | 14.29            | 12.91 | 12.57 | 14.95 | 11.75 | 11.56 | 11.34 | 11.73 | 10.77 | 10.12 | 10   | 9.69               | 8.66 | 9.62 | 9.71 | 9.51 |

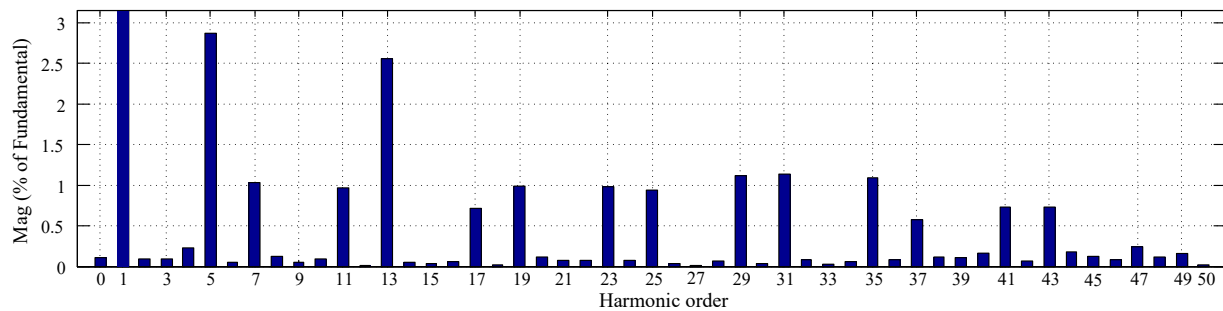
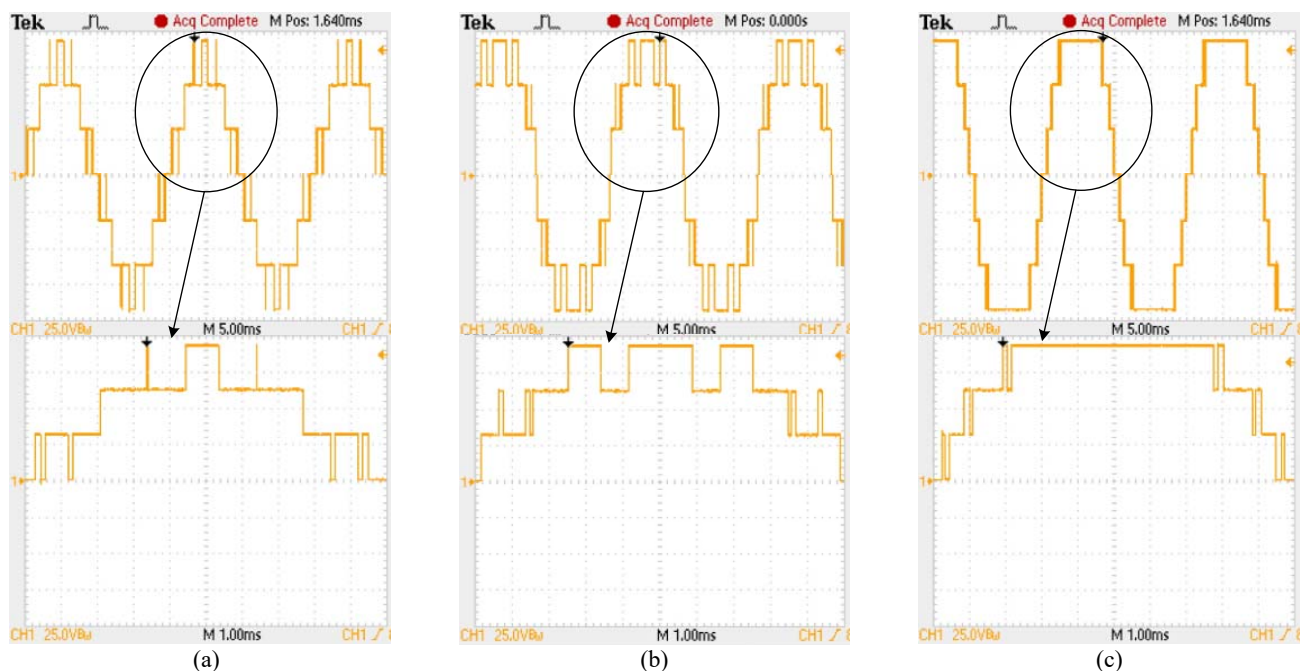
Fig. 9. Harmonic spectra of the experimental results  $m_a=2.7$ .Fig. 10. Experimental investigation of the proposed method in synthesizing SHM-PWM waveform: (a)  $m_a=1.9$ ; (b)  $m_a=2.4$ ; (c)  $m_a=2.745$ .

TABLE VI  
THD AND HARMONIC COMPONENTS OF THE EXPERIMENTAL  
RESULTS FOR THE PROPOSED SHM-PWM METHOD

| Harmonic Order      | 1.7   | 1.9   | 2.2   | 2.4  | 2.5   | 2.7  | 2.745 |
|---------------------|-------|-------|-------|------|-------|------|-------|
| 5                   | 3.93  | 1.61  | 1.58  | 2.29 | 2.86  | 2.81 | 0.5   |
| 7                   | 5     | 2.26  | 4.59  | 2.26 | 2.7   | 1.06 | 0.17  |
| 11                  | 2.74  | 1.79  | 0.72  | 0.3  | 0.74  | 0.94 | 0.34  |
| 13                  | 3     | 1.22  | 3     | 1.06 | 2.01  | 2.53 | 2.76  |
| 17                  | 0.47  | 0.21  | 0.45  | 0.4  | 0.37  | 0.68 | 0.59  |
| 19                  | 1.37  | 0.99  | 0.17  | 0.48 | 1.39  | 0.99 | 0.62  |
| 23                  | 0.76  | 1.31  | 1.14  | 1.26 | 1.33  | 0.98 | 0.17  |
| 25                  | 0.97  | 0.96  | 0.92  | 1.24 | 1.15  | 0.93 | 0.23  |
| 29                  | 0.97  | 0.51  | 0.93  | 0.05 | 1.27  | 1.11 | 0.22  |
| 31                  | 0.98  | 1.15  | 1.08  | 0.63 | 0.24  | 1.18 | 0.15  |
| 35                  | 0.3   | 0.32  | 0.98  | 0.16 | 0.9   | 1.1  | 0.56  |
| 37                  | 0.17  | 0.44  | 0.8   | 0.26 | 0.77  | 0.57 | 0.56  |
| 41                  | 0.93  | 0.64  | 0.48  | 0.62 | 0.73  | 0.68 | 0.37  |
| 43                  | 0.51  | 0.83  | 0.52  | 0.16 | 0.25  | 0.68 | 0.94  |
| 47                  | 0.19  | 0.89  | 0.38  | 0.69 | 0.68  | 0.2  | 0.15  |
| 49                  | 0.11  | 0.58  | 0.18  | 0.66 | 0.08  | 0.15 | 0.57  |
| THD <sup>40th</sup> | 7.9   | 4.23  | 6.25  | 3.94 | 5.32  | 4.87 | 3.08  |
| THD <sup>50th</sup> | 7.98  | 4.48  | 6.3   | 4.11 | 5.41  | 4.97 | 3.3   |
| THD                 | 14.62 | 11.89 | 11.42 | 8.88 | 10.65 | 9.81 | 8.65  |

## VI. CONCLUSIONS

In this paper, an improved optimal low frequency SHM-PWM modulation technique was proposed. In the proposed method, the switching angle constraints are enlarged than conventional SHM-PWM for the H-bridge cells. This option gives more degrees of freedom to the math solver of non-linear equations. Hence, a larger set of answers can be found over a wider range of modulation indices. The proposed SHM-PWM method can successfully mitigate low order harmonics up to the 50<sup>th</sup> term. In addition, it satisfies the grid codes EN 50160 and CIGRE-WG 36-05. By using the proposed switching constraint, the range of the modulation index is improved by almost 30 percent, without increasing the switching frequency of the CHB converter. Moreover, the proposed method shows a better distribution of the power losses among the H-bridge cells.

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