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New Approach for Stability of Perturbed DC-DC Converters

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ABSTRACT

In this paper, a simple technique is presented for robust stability testing of perturbed DC-DC converters having multi-linear uncertainty structure. This technique provides a necessary and sufficient condition for testing robust stability. It is based on the corollary of Routh criterion and gridding of parameters. The previous work based on parametric control theory using Kharitonov's theorem and Hermite Biehler theorem gives conservative results and only the sufficient condition of stability [1-5,16-17], whereas the proposed method provides the necessary and sufficient condition for testing robust stability and it is computationally efficient. The superiority of the method is compared with the Edge theorem [6].

Keywords: Push-pull dc-dc converter, Kharitonov's theorem, Routh criterion, gridding of parameters, Edge theorem, and aerospace applications.

1. Introduction

Switch-mode DC-DC converters are used to convert the unregulated dc-dc input into a controlled dc output at a desired voltage level. They are widely used in computer hardware, aerospace applications, VLSI design and in modern power supplies [7-9]. A PWM Buck DC-DC converter is used to step down the voltage level of a power source. An active clamp push-pull converter is useful in battery sourcing applications [10]. Hence, the robust stability analysis of such type of converters is extremely important. It can be done by using Kharitonov's theorem [11]. Using this method, only four polynomials have to be generated from the parameter variations within the maximum and minimum levels. The Routh's stability test is performed on only these four polynomials to declare the system stability under parameter variations. Furthermore, an important corollary of Kharitonov's theorem is reported for robust stability analysis of low-order uncertain systems [12]. It is shown that to check the stability of low-order interval polynomials, specifically for $n = 3, 4$ and 5, the number of Kharitonov's polynomials required to check robust stability is one, two and three respectively, instead of four. In [1], design of a PWM push-pull converter with state feedback control model is presented, and its robust stability testing has been done using Kharitonov's theorem. Similarly, we have found that in various dc-dc converter applications [2-5], robust stability has been decided by using the parametric control theory based on Kharitonov's theorem. The

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application of the mapping theorem which uses stability techniques, as reported in [13, 14], are also useful in testing the robust stability of the converter [15]. In [16], an improved stability testing technique is shown. It explains that the robust stability analysis of a PWM push-pull converter can be done using simple frequency domain conditions based on the Hermite Biehler theorem [17]. However, it is applicable for low-order uncertain systems and gives only a sufficient condition of stability.

The most efficient criterion is Kharitonov's theorem for testing the robust stability of systems when the uncertainties in the coefficients of the characteristic polynomial are independent. It has been reported that when there is a multi-linear dependent uncertainty structure, Kharitonov's theorem gives conservative results and only a sufficient condition of stability [18-19]. Similarly, when there is an over-bounding of the coefficients, robust parametric control theory (RPCT) i.e., Generalized Kharitonov's theorem [20] also gives conservative results as reported in [4-5]. In certain instances, Kharitonov's theorem may show instability, while in an actual sense, the system is stable. For multi-linear uncertainty, the Edge theorem due to Barlett, Hollot and Lin can be useful [6]. But when the uncertain parameters are large in numbers, a higher number of vertex polynomials are needed. Moreover, the technique is graphical, so even high-speed computers need more time to plot the roots of the polytopic family in the case of Edge theorem and a larger number of images in case of Mapping theorem. Similarly, by using the Nyquist plot and gridding uncertain parameters, robust stability testing of uncertain systems can done as reported in [19]. But, it is also computationally expensive. Since the converters are useful in aerospace applications we need a control technique, which should be simple, computationally efficient and provides a necessary and sufficient condition for testing the robust stability of systems.

In this paper, a new approach based on Kharitonov's theorem and Routh's criterion is presented to obtain a necessary and sufficient condition for the robust stability testing of DC-DC converters. It is based on Krishnamurty's result on Routh's criterion [21] and gridding of uncertain parameters. In order to show the effectiveness of this method, the robust stability analysis of a PWM push-pull DC-DC converter with state feedback [1] was carried out using the proposed approach instead of using only Kharitonov's theorem. The results are verified using the Edge theorem. The proposed analysis is simple and can be useful for practicing engineers.

2. Kharitonov's theorem [11]

The polynomials

$$K(s)=\sum_{i=0}^{n} a_i s^i,\ a_i \in [\alpha_i, \beta_i], \tag{1}$$

where, the real coefficients a_i takes any arbitrary value in the closed interval $[\alpha_i, \beta_i]$ are strictly Hurwitz if and only if the following four polynomials are strictly Hurwitz:

$$k_1(s)=\alpha_0+\beta_1 s+\beta_2 s^2+\alpha_3 s^3+\alpha_4 s^4+\ldots \tag{2}$$

$$k_2(s)=\alpha_0+\alpha_1 s+\beta_2 s^2+\beta_3 s^3+\alpha_4 s^4+\ldots \tag{3}$$

$$k_3(s)=\beta_0+\alpha_1 s+\alpha_2 s^2+\beta_3 s^3+\beta_4 s^4+\ldots \tag{4}$$

$$k_4(s)=\beta_0+\beta_1 s+\alpha_2 s^2+\alpha_3 s^3+\beta_4 s^4+\ldots. \tag{5}$$

Corollary1 [12]: The robust stability of low-order uncertain systems, particularly, $n=3,4$ and 5 can be decided by testing only one, two and three of Kharitonov's polynomials, respectively.

Corollary 2 [21]: The stability of the linear system can be determined testing the s^1 row of the Routh array. If the element in the s^1 of row of the Routh array is zero, then the system is marginally stable and if it is less than zero, then the system is unstable. For stability, it must be greater than zero.

3. Steps for Checking Robust Stability of Multilinear Uncertain Systems

Step 1: Determining all the minimum and the maximum values of the uncertain coefficients using the over-bounding technique.

Step 2: Formulation of all four of Kharitonov's polynomials using coefficients from Step 1.

Step 3: Checking all four of the polynomials using the Routh criterion except only one, two and three polynomials when the order of the system is $n = 3, 4$ and 5, respectively using Kharitonov's theorem.

Step 4: If all four polynomials are Hurwitz, then the system is stable, which is a necessary and sufficient condition for robust stability. But, if any of the polynomials in Step 3 are not Hurwitz, then we can not say that the system is unstable. Therefore, corollary based testing using the Routh criterion and gridding of uncertain parameters can be useful, which are shown in the following steps.

Step 5: Formulating the Routh array for the uncertain system and then using Krishmurthi's results on Routh criterion, a necessary and sufficient condition (RC) is established for the elements in the s^1 row of the Routh array. For robust stability,

$$RC > 0 \quad . \tag{6}$$

Step 6: By choosing some points within each interval of the uncertain elements, the condition in Step 5 can be checked. For example, suppose five points are chosen within each interval of the uncertain elements, say t_1, t_2 and t_3, then a total of $5\times5\times5 = 125$ conditions need to be checked.

Step 7: If all these conditions are satisfied, then the uncertain system is completely stable.

4. PWM Push-Pull DC- DC Converter with State Feedback Control

The push-pull converter is one of the most important multiple switch isolated DC-DC converters. It is useful in various applications like aerospace where in the robust stability analysis of such types the converter is extremely important. Detailed explanations of the equivalent circuit with parasitic components of the PWM push-pull DC-DC converter and its modeling are given in [1]. In [1], using the multivariable integral control technique, the complete design of the linearized PWM push-pull DC-DC converter with state feedback control has been presented in terms of a linear block diagram model. The same block diagram is shown in Fig. 1, where V_{ref} is the desired steady state output of the system and at $t \to \infty, v_0 \to V_{ref}$. L, C, r, r_C are the inductor, capacitor, equivalent resistance in series with the inductor and equivalent series resistance of the capacitor respectively. k_i, k_v and k_y are the current, voltage and output feedback-loop gains respectively, which were calculated using the pole-placement technique. The other details of the block diagram are given in [1].

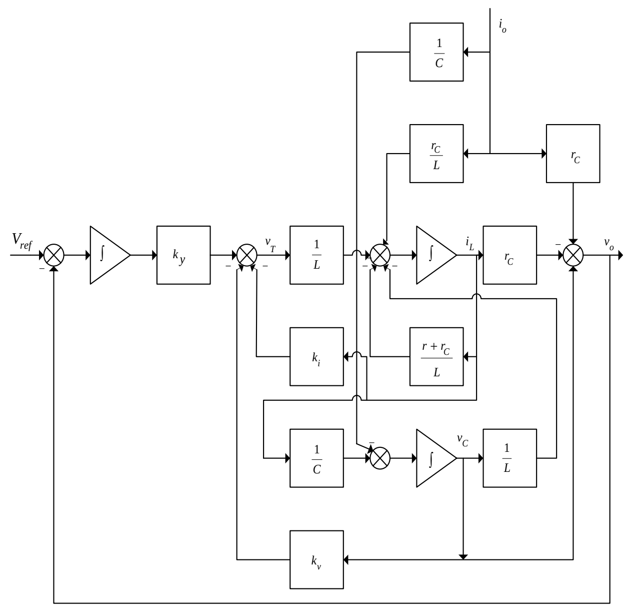

Fig. 1 Block diagram of the linearized PWM DC-DC converter with a state-feedback control

From Fig. 1, the closed loop characteristic equation of the closed loop PWM DC-DC converter with integral control is

$$s^3 + \frac{r + rc + k_i}{L}s^2 + \frac{r_C CK_y + k_v + 1}{LC}s + \frac{k_y}{LC}. \tag{7}$$

5. Robust Stability Testing OF PWM Push-Pull DC-DC Converter

The robust stability testing of a PWM push-pull DC-DC converter system with state feedback has been presented in [1] using Kharitonov's theorem. It is specified that the components and the gains of the system are subject to the following uncertainties, i.e.,

$$r = 0.330 \pm 0.2\Omega, r_C = 0.04 \pm 0.02\Omega,$$
$$L = (10 \pm 2)\times 10^{-6}, C = (47 \pm 20)\times 10^{-6} F, \tag{8}$$
$$k_i = 7.63 \pm 1\Omega, k_v = 77.7 \pm 10H,$$
$$k_y = 5.64\times 10^6 \pm 5\times 10^5$$

When using Kharitonov's theorem for these uncertainties as shown in eq. (8), the system is stable. Now the uncertainties shown in eq. (8), particularly, k_v and k_y are modified, which are as shown in eq. (9).

$$r = 0.330 \pm 0.2\Omega, r_C = 0.04 \pm 0.02\Omega,$$
$$L = (10 \pm 2)\times 10^{-6} H, C = (47 \pm 20)\times 10^{-6} F, \tag{9}$$
$$k_i = 7.63 \pm 1\Omega, k_v = 77.7 \pm 40,$$
$$k_y = 5.64\times 10^6 \pm 10\times 10^5 s^{-1}.$$

We checked the robust stability of the PWM push-pull DC-DC converter using the steps mentioned in section- 3. It is as follows.

Step 1: Consider the polynomial,

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0, \tag{10}$$

in which the ranges of the coefficients are,

$$a_0 \in [x_0, y_0], a_1 \in [x_1, y_1], a_2 \in [x_2, y_2], a_3 = 1. \tag{11}$$

Comparing eq. (7) and eq. (10), we determine all the uncertain values as shown in step 2.

Step 2: The uncertain values for all the coefficients of eq. (10) are

$$x_2 = \min\left\{\frac{r + r_C + k_i}{L}\right\} = 5.6500\times 10^5,$$
$$y_2 = \max\left\{\frac{r + r_C + k_i}{L}\right\} = 1.1525\times 10^6,$$
$$x_1 = \min\left\{\frac{r_c C k_y + k_v + 1}{LC}\right\} = 5.1251\times 10^{10},$$
$$y_1 = \max\left\{\frac{r_c C k_y + k_v + 1}{LC}\right\} = 6.7311\times 10^{11}, \tag{12}$$
$$x_0 = \min\left\{\frac{k_y}{LC}\right\} = 5.7711\times 10^{15},$$
$$y_0 = \max\left\{\frac{k_y}{LC}\right\} = 3.074\times 10^{16}.$$

Step 3: We formulate all four of Kharitonov's polynomials;

$$k_1(s) = s^3 + 5.65\times 10^5 s^2 + 5.1251\times 10^{10} s + 3.074\times 10^{16} \tag{13}$$
$$k_2(s) = s^3 + 5.65\times 10^5 s^2 + 6.7311\times 10^{11} s + 3.074\times 10^{16} \tag{14}$$
$$k_3(s) = s^3 + 1.1525\times 10^6 s^2 + 5.1251\times 10^{10} s + 5.7711\times 10^{15} \tag{15}$$
$$k_4(s) = s^3 + 1.1525\times 10^6 s^2 + 6.7311\times 10^{11} s + 5.7711\times 10^{15}. \tag{16}$$

Step 4: Since the order of the system is three, using corollary 1, it is necessary to apply the Routh criterion to polynomial $k_1(s)$ only. The roots of the polynomial $k_1(s)$ are $-5.6975\times 10^5, (0.0237 \pm 2.3227)\times 10^5$. Since some of the roots of polynomial $k_1(s)$ are lying on the right half of the s-plane the system, i.e. the PWM push-pull DC-DC converter, is unstable. If the system is found to be stable using Kharitonov's theorem, then it has a necessary and sufficient condition of stability. But, in this case, the system is unstable; hence we can not say that the actual system is unstable. Next we checked the actual robust stability using Routh criterion and gridding technique as shown in the following steps.

Step 5: By applying Routh criterion to the polynomials eq. (7), the following Routh array is obtained,

$$\begin{array}{lll} s^3 & 1 & \dfrac{r_C C k_y + k_v + 1}{LC} \\ s^2 & \dfrac{r + r_C + k_i}{L} & \dfrac{k_y}{LC} \end{array}$$

(Common factor $\frac{1}{L}$ is eliminated)

$$\begin{array}{lll} s^2 & r + r_C + k_i & \dfrac{k_y}{C} \\ s^1 & \multicolumn{2}{l}{\dfrac{(r + r_C + k_i)\times\left(\dfrac{r_C C k_y + k_v + 1}{LC}\right) - (1)\times\left(\dfrac{k_y}{C}\right)}{(r + r_C + k_i)}} \\ s^0 & \dfrac{k_y}{C}. & \end{array}$$

Using corollary 2 of the Routh criterion, the value in the s^1 row should be greater than zero. So, we get,

$$\frac{(r + r_C + k_i)\times\left(\dfrac{r_C C k_y + k_v + 1}{LC}\right) - (1)\times\left(\dfrac{k_y}{C}\right)}{(r + r_C + k_i)} > 0 \tag{17}$$

From eq. (17), we get the following robust stability condition,

$$RC = (r + r_c + k_i)(r_c C k_y + k_v + 1) - (L k_y) \tag{18}$$

Step 6: Now we grid the parameters using say, 3 values within each interval of r, r_c, L, C, k_i, k_v and k_y, i.e., 7 uncertainties. Thereafter, using MATLAB, 343 (3^7), robust stability conditions (RC) are checked. It is found that for all the 343 conditions,

$$RC > 0, \tag{19}$$

and hence the uncertain PWM push-pull DC-DC converter is stable. But when using Kharitonov's theorem, as shown in step 4, the system is found to be unstable. The results are verified by using the Edge theorem [6]. Using the Edge theorem, the boundary of the root loci of the interval polynomial family can be obtained by plotting the root loci along the exposed edges of the box. The number of vertices and exposed edges are given by 2^n and $n2^{n-1}$ respectively, where n is the number of uncertain parameters. In this case, the number of uncertain parameters is 7, hence 2^7 vertex polynomials are needed. So, using 128 vertex polynomials, 448 edges are checked using the Root locus technique. The root loci of all the edges are shown in Fig. 2. Since the entire root space of the set of characteristic polynomials is found to be in the open left half of the s-plane. Hence, the PWM push-pull DC-DC converter with state feedback system is robustly stable. The comparative results are shown in tabular form in Table 1.

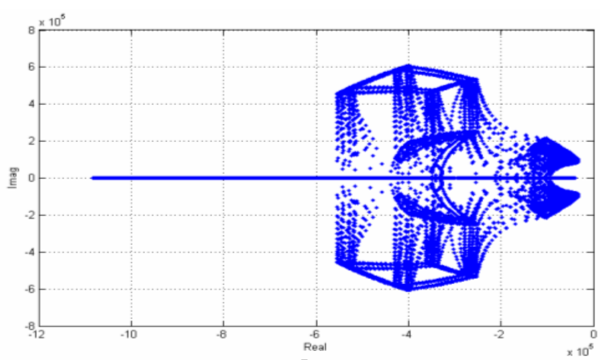

Fig. 2 Robust stability testing of PWM DC-DC converter using Edge theorem

Table 1 Comparison between various stability testing methods for robust stability of PWM push-pull DC-DC converter with state feedback

Various methods for testing stability of PWM push-pull DC-DC converter.	Computational time using tic-toc command in MATLAB using Pentium-IV processor.	Remarks
D. Czarkowski's et.al. approach, using Kharitonov's theorem [1].	0.01 second	(i) All the four Kharitonov's polynomials need to be checked. (ii) It only gives sufficient condition of stability.
Edge theorem By A.C. Bartlett et.al.[6]	41.84 second	(i) It gives necessary and sufficient condition of stability. (ii) It is computationally expensive.
Proposed method	3.70 second	(i) Only one Kharitonov's polynomial needs to be checked. (ii) It is simple and computationally inexpensive. (iii) It gives necessary and sufficient condition of stability.

6. Conclusions

Kharitonov's theorem gives only a sufficient condition for testing robust stability. Hence, using only Kharitonov's theorem can not be useful for robust stability analysis of DC-DC converters, particularly, in aerospace applications. Hence the proposed technique based on Krishnamurthy's result on Routh's criterion and gridding technique can be useful, by providing a necessary and sufficient condition for stability testing. The proposed approach is simple and computationally efficient in comparison with existing methods such as the Edge theorem and Mapping theorem.

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