# Analytical Model of Conduction and Switching Losses of Matrix-Z-Source Converter 

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#### Abstract

This paper investigates analytical models of Conduction and Switching Losses (CASLs) of a matrix-Z-source converter (MZC). Two analytical models of the CASLs are obtained through the examination of operating principles for a Z-source inverter and ac-dc matrix converter respectively. Based on the two models, the analytical model of CASLs for a MZC is constructed and visualized over a range of exemplified operating- points, each of which is defined by the combination of power factor $(\mathrm{pf})$ and modulation index $(M)$. The model provides a measurable way to approximate the total losses of the MZC.


Keywords: Conduction and Switching Losses, Matrix-Z-Source Converter, Analytical model of loss

## 1. Introduction

A matrix-Z-source converter (MZC) for bidirectional three-phase ac-dc power conversion was proposed marrying up both the advantages of the matrix converter and the $Z$-source inverter ${ }^{[1]}$. For a practical engineering development, MZC needs the investigation of conduction-and-switching-losses (CASLs) caused by its power-switch network. Nevertheless, the CASLs are essential measures in the comparison of merits between the MZC and existing bidirectional three-phase ac-dc power converters. The study of CASLs for MZC, therefore, is desirable and will be presented through analytical modeling in this paper.

The study of CASLs of a given power converter is usually carried out through a specific methodology using

[^0]two elements, the operating principle of the converter and the CASLs model of each power-switch ${ }^{[2]-[9]}$. Section 2 briefly describes the operating principles of MZC, and Section 3 introduces the methodology of modeling CASLs and presents the parameters of CASLs for IGBT and FreeWheeling Diode (FWD) as the base of modeling. Subsequently, sections 4 and 5 investigate the models of CASLs of Z-source inverter and ac-dc matrix converter respectively; section 6 then sets up the complete model of CASLs of MZC by using the two aforesaid models in sections 4 and 5.

## 2. Topology and Modulation Strategies of the Matrix-Z-source Converter

Detail Configuration of MZC may be found in [1] and is not reprinted here for brevity. MZC has two operating modes, the dc-ac inversion mode and the ac-dc rectification mode. Operating principles of each mode are briefly
described as follows.

### 2.1 Brief description of the operating principle of the MZC in dc-ac inversion mode

MZC has the same configuration as the Z-source inverter when MZC is in dc-ac inversion mode ${ }^{[1]}$. The Zsource inverter has the same structure of a power-switch network as that of VSI but with a different operating principle ${ }^{[10]}$. Fig. 1 shows the basic structure of a $Z$-source inverter. Its modeling and modulation strategy is given by (1)-(5) derived from Alesina-Venturini optimum PWM (AV-optimum PWM ${ }^{[12] \text { and }[14]}$. In this modeling Maximum Constant Boost Control (MCBC) is employed ${ }^{[1]}$. Details of MCBC may be found in [11]. Fig. 4 in section 4 shows an example of the gate-drive logic and corresponding currents within one switching cycle.


Fig. 1 Basic structure of Z-source inverter used in MZC when MZC is in dc-ac inversion mode

$$
\begin{align*}
& {\left[\begin{array}{l}
V_{a}(t) \\
V_{b}(t) \\
V_{c}(t)
\end{array}\right]=\left[\begin{array}{ll}
d_{p a} & d_{n a} \\
d_{p b} & d_{n b} \\
d_{p c} & d_{n c}
\end{array}\right] B\left[\begin{array}{l}
V_{d c} / 2 \\
-V_{d c} / 2
\end{array}\right]} \\
& \frac{1}{B}\left[\begin{array}{l}
I_{p} \\
-I_{p}
\end{array}\right]=\left[\begin{array}{lll}
d_{p a} & d_{p b} & d_{p c} \\
d_{n a} & d_{n b} & d_{n c}
\end{array}\right]\left[\begin{array}{l}
I_{a}(t) \\
I_{b}(t) \\
I_{c}(t)
\end{array}\right]  \tag{1}\\
& d_{p a}=M[(1 / 2) \cos (\omega t)-(1 / 12) \cos (3 \omega t)]+1 / 2 \\
& d_{p b}=M[(1 / 2) \cos (\omega t-2 \pi / 3)-(1 / 12) \cos (3 \omega t)]+1 / 2 \\
& d_{p c}=M[(1 / 2) \cos (\omega t+2 \pi / 3)-(1 / 12) \cos (3 \omega t)]+1 / 2 \\
& d_{n a}=1-d_{p a} \\
& d_{n b}=1-d_{p b} \\
& d_{n c}=1-d_{p c} \tag{2}
\end{align*}
$$

$$
\begin{equation*}
B=1 /(\sqrt{3} M-1) \quad 1 / \sqrt{3}<M<2 / \sqrt{3} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
D_{0}=1-\frac{\sqrt{3}}{2} M \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
V_{p n}=B V_{d c}=\frac{1}{1-2 D_{0}} V_{d c}=\frac{1}{\sqrt{3} M-1} V_{d c} \tag{5}
\end{equation*}
$$

where

$$
\begin{aligned}
& {\left[\begin{array}{c}
V_{a}(t) \\
V_{b}(t) \\
V_{c}(t)
\end{array}\right] }=\left[\begin{array}{l}
V_{p k} \cos (\omega t) \\
V_{p k} \cos (\omega t-2 \pi / 3) \\
V_{p k} \cos (\omega t+2 \pi / 3)
\end{array}\right] \\
& {\left[\begin{array}{l}
I_{a}(t) \\
I_{b}(t) \\
I_{c}(t)
\end{array}\right]=\left[\begin{array}{l}
I_{p k} \cos (\omega t+\Phi) \\
I_{p k} \cos (\omega t-2 \pi / 3+\Phi) \\
I_{p k} \cos (\omega t+2 \pi / 3+\Phi)
\end{array}\right] }
\end{aligned}
$$

$M$ is the modulation index; $d_{i j}$ is the duty-cycle of switch $S_{i j}(i=p, n, j=a, b, c) ; V_{p k}$ and $I_{p k}$ are peak phase voltage and peak current respectively; $I_{p}$ is the average dc-link current; $V_{d c}$ is the dc source voltage; $\omega$ is the fundamental electric angular frequency; $\Phi$ is ac current phase angle; $B$ is the boost factor; $D_{0}$ is shoot-through duty-cycle; $V_{p n}$ is the peak dc-link voltage.

### 2.2 Brief description of the operating principle of the MZC in ac-dc rectification mode

MZC has the came configuration as ac-dc matrix when MZC is in the ac-dc rectification mode ${ }^{[1]}$ as shown in Fig. 2. The operating principles using AV-optimum PWM may be described in (6)-(8) ${ }^{[1] ~ a n d ~[13]}$. Fig. 5 and Fig. 6 in section 5 show an exemplified procedure of current commutation .


Fig. 2 Basic structure of ac-dc matrix converter used in MZC when MZC is in ac-dc rectification mode

$$
\left[\begin{array}{l}
V_{d c} / 2  \tag{6}\\
-V_{d c} / 2
\end{array}\right]=\left[\begin{array}{lll}
d_{p a} & d_{p b} & d_{p c} \\
d_{n a} & d_{n b} & d_{n c}
\end{array}\right]\left[\begin{array}{l}
V_{a}(t) \\
b_{b}(t) \\
V_{c}(t)
\end{array}\right] ;\left[\begin{array}{l}
I_{a}(t) \\
I_{b}(t) \\
I_{c}(t)
\end{array}\right]=\left[\begin{array}{ll}
d_{p a} & d_{n a} \\
d_{p b} & d_{n b} \\
d_{p c} & d_{n c}
\end{array}\right]\left[\begin{array}{c}
I_{d c} \\
-I_{d c}
\end{array}\right]
$$

$d_{p a}=M\left[\begin{array}{l}(1 / 2) \cos \left(\omega_{\text {in }} t\right)+ \\ (7 / 36) \cos \left(2 \omega_{\text {in }} t\right)-(1 / 36) \cos \left(4 \omega_{\text {in }} t\right)\end{array}\right]+1 / 3$
$d_{p b}=M\left[\begin{array}{l}(1 / 2) \cos \left(\omega_{\text {in }} t-2 \pi / 3\right)+ \\ (7 / 36) \cos \left(2 \omega_{\text {in }} t+2 \pi / 3\right)-(1 / 36) \cos \left(4 \omega_{\text {in }} t\right)\end{array}\right]+1 / 3$
$d_{p c}=1-d_{p a}(t)-d_{p b}(t)$
$d_{\text {na }}=M\left[\begin{array}{l}(-1 / 2) \cos \left(\omega_{\text {in }} t\right)+ \\ (7 / 36) \cos \left(2 \omega_{\text {in }} t\right)-(1 / 36) \cos \left(4 \omega_{\text {in }} t\right)\end{array}\right]+1 / 3$
$d_{n b}=M\left[\begin{array}{l}(-1 / 2) \cos \left(\omega_{\text {in }} t-2 \pi / 3\right)+ \\ (7 / 36) \cos \left(2 \omega_{\text {in }} t+2 \pi / 3\right)-(1 / 36) \cos \left(4 \omega_{i n} t\right)\end{array}\right]+1 / 3$
$d_{n c}=1-d_{n a}(t)-d_{n b}(t)$
$0<M=\frac{4 V_{\text {dc }}}{3 \sqrt{3} V_{i n}}<2 / \sqrt{3}$
Where,

$$
\begin{aligned}
& {\left[\begin{array}{l}
V_{a}(t) \\
V_{b}(t) \\
V_{c}(t)
\end{array}\right]=\left[\begin{array}{l}
V_{i n} \cos \left(\omega_{\text {in }} t\right) \\
V_{i n} \cos \left(\omega_{\text {in }} t-2 \pi / 3\right) \\
V_{i n} \cos \left(\omega_{\text {in }} t+2 \pi / 3\right)
\end{array}\right] ;} \\
& {\left[\begin{array}{l}
I_{a}(t) \\
I_{b}(t) \\
I_{c}(t)
\end{array}\right]=\left[\begin{array}{l}
I_{i n} \cos \left(\omega_{i n} t+\Phi\right) \\
I_{i n} \cos \left(\omega_{i n} t-2 \pi / 3+\Phi\right) \\
I_{i n} \cos \left(\omega_{i n} t+2 \pi / 3+\Phi\right)
\end{array}\right] ;}
\end{aligned}
$$

$V_{i n}$ and $I_{i n}$ are peak phase voltage and peak current respectively; $\omega_{i n}$ is ac angular speed; $\Phi$ is input ac current phase angle; $V_{d c}$ and $I_{d c}$ are the output dc voltage and dc current respectively; $d_{i j}(i=p, n ; j=a, b, c)$ is duty-cycle.

## 3. Method of Modeling CASLs and Experimental Data to be Used

### 3.1 Methodology of modeling CASLs of power converters

The methodology may be grouped into two categories: one for dynamic electro thermal behavior and one for steady-state CASLs. The methodology for dynamic electrothermal behavior focuses on the power-switches in a given power converter; the relevant CASLs model of the power-switch is a physical-based model where the junction temperature is treated as a variable. Due to the complexity of the physical-based models, this methodology relies on simulation tools and has to focus on the study at a single operating-point ${ }^{[2],[3]}$. In the cases where the quick estimation of the CASLs of a power converter over a range of operating-points is required, this methodology may theoretically be used but will lead to an over complicated procedure. On the other hand, the methodology for steady-
state CASLs focuses on the total CASLs of a power converter; the relevant CASLs model of the power-switch consists of an equivalent conduction-loss model and an equivalent switching-loss model. The junction temperature in this methodology is treated as a given unchangeable condition. Due to its simplicity, this methodology does not rely on simulation tools and is able to analytically express the approximated CASLs of a power converter over a range of operating-points ${ }^{[4], ~[9]}$. Within this methodology, on-state voltage/resistance and turn-on/off switching energy at a given junction temperature are critical to approximate the conduction loss and switching loss respectively. A powerful and fairly accurate way is to measure these parameters and then describe the dependency of the conduction loss or the switching loss on the applied voltage and/or current in a simple equation. The equation for the conduction loss is obvious. The equation for the switching loss, however, is obtained through various ways of approximation ${ }^{[4]-[6], ~[8]}$. A set of the measured on-state voltage/resistance and a quadratic leastsquare approximation of the dependency of the switching energy on the switched voltage/current were reported in [6] and verified through ${ }^{[7]}$. The experimental data will be employed in this paper where the quick estimation of CASLs of MZC over a range of operating-points is required.

### 3.2 Conduction-loss model approximation

Fig. 3 shows the model of IGBT or FWD as the combined voltage source and resistor in series. The tested parameters reported in [6] are listed in Table 1.


Fig. 3 Equivalent circuit of IGBT or FWD for calculating the conduction loss, where $V_{T}$ is voltage drop of IGBT; $V_{D}$ is forward voltage drop of FWD; $r_{T}$ and $r_{D}$ are on-state resistances of IGBT and free-wheel diode respectively

Table 1 Tested On-state parameters of IGBT and FWD at 120
${ }^{\circ} \mathrm{C}$ of junction temperature

| $\boldsymbol{T}_{\boldsymbol{j}}$ | $\boldsymbol{r}_{\boldsymbol{T}}$ | $\boldsymbol{r}_{\boldsymbol{D}}$ | $\boldsymbol{V}_{\boldsymbol{T}}$ | $\boldsymbol{V}_{\boldsymbol{D}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 2 0}^{\circ} \mathbf{C}$ | 0.0787 | 0.038 | 0.768 | 0.732 |
| Units | V/A | V/A | V | V |

The switch devices under discussion are gated on/off by ideal PWM signals. The conduction-loss model within every switching cycle can be approximated as given by (9).

$$
\begin{equation*}
p_{c}=r_{T / D} i^{2}{ }_{T / D r m s}+v_{T / D}\left\langle i_{T / D}\right\rangle \tag{9}
\end{equation*}
$$

where $i_{T / D r m s}$ is the R.M.S. current within the switching cycle; $\left\langle i_{T I D}\right\rangle$ is the average current within the switching cycle.

### 3.3 Switching-loss model approximation

The Switching-loss model is obtained through averaging total energy losses caused by individual turn-on/off actions in a switching cycle. The dependency of the individual energy loss, $w$, on the switched voltage and switched current is given by (10) (11) (12). The turn-on energy loss of a diode is ignored.
$w_{\text {Toff }}=K_{\text {Toff } 1} u i+K_{\text {Toff } 2} u i^{2}+K_{\text {Toff } 3} u^{2}+K_{\text {Toff } 4} u^{2} i+K_{\text {Toff } 5} u^{2} i^{2}=w_{\text {Toff }}(u, i)$
$w_{\text {Ton }}=K_{\text {Ton1 }} u i+K_{\text {Ton } 2} u i^{2}+K_{\text {Ton3 } 3} u^{2}+K_{\text {Ton4 }} u^{2} i+K_{\text {Ton5 }} u^{2} i^{2}=w_{\text {Ton }}(u, i)$
$w_{\text {Doff }}=K_{\text {Don } 1} u i+K_{\text {Don } 2} u i^{2}+K_{\text {Don } 3} u^{2}+K_{\text {Don } 4} u^{2} i+K_{\text {Don } 5} u^{2} i^{2}=w_{\text {Doff }}(u, i)$
where $u$ and $i$ are the switched voltage and switched current respectively; $T$ and $D$ denote IGBT and FWD respectively; $K_{i}(i=1,2, \ldots 5)$ is the coefficient for approximation listed in Table $2{ }^{[6]}$.

Table 2 Coefficients of the least-square approximation of the measured IGBT/FWD switching losses at $120^{\circ} \mathrm{C}$ junction temperature $\left(T_{j}\right)^{[6]}$

| $T_{j}$ |  | $K_{1}$ | $\mathrm{K}_{2}$ | $K_{3}$ | $\mathrm{K}_{4}$ | $K_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 120^{\circ} \\ & \mathrm{C} \end{aligned}$ | $T_{\text {off }}$ | 179 | $1.31$ | 0.650 | -0.116 | $\begin{gathered} 0.0034 \\ 8 \end{gathered}$ |
|  | $T_{\text {on }}$ | 70.0 | 2.94 | 0.518 | 0.102 | $\begin{gathered} 0.0015 \\ 5 \end{gathered}$ |
|  | $D_{\text {off }}$ | 97.7 | $3.73$ | 0.488 | 0.140 | $\begin{gathered} 0.0042 \\ 7 \end{gathered}$ |
| Units |  | nWs $(\mathrm{VA})^{-1}$ | (VW $\begin{array}{r}\mathrm{nWs} \\ \left(\mathrm{VA}^{-1}\right.\end{array}$ | $\left.{ }^{\text {nW }}{ }^{2}\right)^{-1}(\mathrm{~V}$ | $\mathrm{nWs}(\mathrm{V}$ $\left.{ }^{2} \mathrm{~A}\right)^{-1}$ | $\begin{aligned} & \mathrm{nWs}(\mathrm{~V} \\ & \left.{ }^{2} \mathrm{~A}^{2}\right)^{-1} \end{aligned}$ |

## 4. Average CASLs Model of the Powerswitch Network of Z-source Inverter

The analytical model of power loss for the $Z$-source inverter is developed in this section. The CASLs of the Zsource inverters is evenly distributed, hence, can be approximated by examining the CASLs of one pair of IGBT and its FWD, i.e. the $T_{p a}$ and $D_{n a}$ shown in Fig. 1, where current $i_{a}$ refers $I_{a}(t)$ of (1). Switching frequency is assumed much higher than the line frequency.

### 4.1 Conduction Loss model of Z-source Inverter

### 4.1.1 Conduction loss Caused by IGBT $T_{p a}$ in Zsource Inverter

When the ac load current $i_{a}$ is in the positive-half cycle, i.e. $i_{a}>0$, within every switching cycle, the current crossing the $T_{p a}$ consists of the shoot-through current and ac load current. Considering Fig. 4, one has (13) and (14) respectively.

$$
\begin{align*}
& \left\langle i_{T p a}\right\rangle=D_{0} \frac{2}{3} I_{L}+d_{p a} i_{a} \\
& i_{\text {Tparms }}^{2}=D_{0}\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right)^{2}+\left(d_{p a}-\frac{D_{0}}{2}\right) i_{a}{ }^{2} \tag{14}
\end{align*}
$$

where $\left\langle i_{T p a}\right\rangle$ is the average current over switching cycle; $i_{\text {Tparms }}$ is the R.M.S current over switching cycle; $D_{0}$ is the shoot-through duty-cycle; $I_{L}$ is the average $Z$-source inductor current; $d_{p a}$ is the duty-cycle of $T_{p a}, i_{a}$ is the ac load current.
In the negative-half cycle, i.e. $i_{a}<0$, only the shootthrough current crosses $T_{p a}$, as that of $i_{\text {Tna }}$ in Fig. 4. The average and R.M.S. currents are

$$
\begin{align*}
& \left\langle i_{\text {Tpa }}\right\rangle=D_{0}\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right)  \tag{15}\\
& i_{\text {Tparms }}{ }^{2}=D_{0}\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right)^{2} \tag{16}
\end{align*}
$$

The average conduction loss caused by $T_{p a}$ over line cycle, therefore, is given by (17)

$$
\begin{align*}
& P_{T P a, c}=\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{3 \pi}{2}+\Phi}\left(r_{T} i^{2}\right. \\
&=r_{T p a m s}+V_{T}\left\langle i_{T p a}\right) d \varphi \\
&\left.\frac{1}{2 \pi} \int_{\frac{\pi}{2}+\Phi}^{\frac{3 \pi}{2}+\Phi} D_{0}\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right)^{2} d \varphi+\frac{1}{2 \pi} \int_{\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi}\left(d_{p a}-\frac{D_{0}}{2}\right) i_{a}^{2} d \varphi\right]  \tag{17}\\
&+V_{T}\left[\frac{1}{2 \pi} \int_{\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi}\left(D_{0} \frac{2}{3} I_{L}+d_{p a} i_{a}\right) d \varphi+\frac{1}{2 \pi} \int_{\frac{\pi}{2}+\Phi}^{\frac{3 \pi}{2}+\Phi} D_{0}\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right) d \varphi\right]
\end{align*}
$$

where $\varphi=\omega t ; i_{a}$ is $I_{a}(t)$ in (1); $D_{0}$ was given by (4).
Replacing variables of $i_{a}, d_{p a}$ and $D_{0}$ in (17). by those in (1), (2) and (4), after some trigonometric operation, the average conduction loss caused by IGBT $T_{p a}$ may be approximated as given by (18), assuming that the power loss is significantly smaller than the active power flow in a proper designed $Z$-source converter.
$P_{\text {pact }}=\left[\begin{array}{l}r_{r}\left(I_{\alpha_{\alpha}}^{2}\left(\frac{1}{8}+\frac{M \cos \Phi}{3 \pi}-\frac{M \cos 3 \Phi}{90 \pi}\right)+I_{t}^{2} \frac{2(2-\sqrt{3} M)}{9}\right)+ \\ V_{r}\left(I_{\mu_{\alpha}}\left(\frac{1}{2 \pi}+\frac{M \cos \Phi}{8}\right)+I_{t}\left(\frac{2-\sqrt{3 M}}{2}\right)\right)\end{array}\right]$
where,

$$
\begin{align*}
I_{L} & =\frac{P_{o}}{V_{d c}}  \tag{19}\\
I_{p k} & =\frac{4}{3} \frac{\sqrt{3} M-1}{M \cos \Phi} \frac{P_{o}}{V_{d c}} \tag{20}
\end{align*}
$$

where $I_{L}$ is the average $Z$-source inductor current; $I_{p k}$ is peak ac load current; $P_{o}$ is the active power; $M, V_{d c}$ and $\Phi$ were defined in (1), (2) and (3).

### 4.1.2 Conduction loss caused by FWD $D_{n a}$ in Zsource Inverter

Only in the positive-half line cycle can the FWD $D_{n a}$ carry load current, as in Fig. 4. The average current and the R.M.S. current of $i_{\text {Dna }}$ over switching cycle is given by (21) and (22).


Fig. 4 Waveforms of $T_{p a}$ and $D_{n a}$ of Fig. 1 in one switching cycle when $i_{a}>0$, where shadowed are shoot-through intervals. $g_{p a}$ and $g_{n a}$ are gate-drive logic of $T_{p a}$ and $T_{n a}$ respectively.

$$
\begin{align*}
& \left\langle i_{\text {Dna }}\right\rangle=\left(1-d_{p a}-\frac{D_{0}}{2}\right) i_{a}=\left(d_{n a}-\frac{D_{0}}{2}\right) i_{a}  \tag{21}\\
& i^{2}{ }_{\text {Dnarms }}=\left(d_{n a}-\frac{D_{0}}{2}\right) i_{a}^{2} \tag{22}
\end{align*}
$$

where $\left\langle i_{\text {Dna }}\right\rangle$ is the average current; $i_{\text {Dnarms }}$ is the R.M.S current; $d_{n a}$ is the duty-cycle of $D_{n a}, i_{a}$ is the ac load current. Therefore, the average conduction loss caused by $D_{n a}$ over line cycle is given by (23).

$$
\begin{align*}
P_{\text {Dna }, c} & =\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi}\left(r_{D} i^{2} \text { Dnarms }^{2}+V_{D}\left\langle i_{\text {Dna }}\right\rangle\right) d \varphi \\
& =r_{D}\left[\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi}\left(d_{n a}-\frac{D_{0}}{2}\right) i_{a}^{2} d \varphi\right]+V_{D}\left[\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi}\left(d_{n a}-\frac{D_{0}}{2}\right) i_{a} d \varphi\right] \tag{23}
\end{align*}
$$

where $\varphi=\omega t ; \Phi$ is ac current phase angle; $i_{a}=I_{a}(t)$. Replacing variables of $d_{n a}, i_{a}$, in (23) by those in (1) and (2), after some trigonometric operation, the average conduction loss caused by free-wheel diode $D_{n a}$ can be approximated as given by (24).
$P_{\text {Dna }, c}=\left[\begin{array}{l}r_{D} I_{p k}{ }^{2}\left(\frac{1-D_{0}}{8}-\frac{M \cos \Phi}{3 \pi}+\frac{M \cos 3 \Phi}{90 \pi}\right)+ \\ V_{D} I_{p k}\left(\frac{1-D_{0}}{2 \pi}-\frac{M \cos \Phi}{8}\right)\end{array}\right]$
where $I_{p k}$ was given by (20).

### 4.1.3 Total average conduction loss in Z-source Inverter

Since the loss is evenly distributed in the power-switch network of the $Z$-source inverter, the total conduction loss can be approximated through (18) and (24) as given by (25).

$$
\begin{align*}
P_{z, c} & =6 \times\left(P_{T p a, c}+P_{D n a, c}\right) \\
& =6 \times\left[\begin{array}{l}
r_{T} I_{p k}^{2}\left(\frac{1}{8}+\frac{M \cos \Phi}{3 \pi}-\frac{M \cos 3 \Phi}{90 \pi}\right)+r_{T}\left(\frac{2}{3} I_{L}\right)^{2} D_{0}+ \\
V_{T} I_{p k}\left(\frac{1}{2 \pi}+\frac{M \cos \Phi}{8}\right)+V_{T} I_{L} D_{0}+ \\
r_{D} I_{p k}^{2}\left(\frac{1-D_{0}}{8}-\frac{M \cos \Phi}{3 \pi}+\frac{M \cos 3 \Phi}{90 \pi}\right)+ \\
V_{D} I_{p k}\left(\frac{1-D_{0}}{2 \pi}-\frac{M \cos \Phi}{8}\right)
\end{array}\right] \tag{25}
\end{align*}
$$

where $D_{0}, I_{L}$ and $I_{p k}$ were given by (4), (19) and (20) respectively.

### 4.2 Switching loss model of Z-source Inverter

### 4.2.1 Switching loss caused by IGBT Tpa in Zsource Inverter

As shown in As shown in, when the ac load current is positive, i.e. $i_{a}>0$, the $T_{p a}$ in the $Z$-source inverter has four switching actions. Two of them, i.e. one turn-on and one turn-off in a non-shoot-through interval, are the same as that in an ordinary VSI, in which the switched voltage is the peak dc-link voltage and the switched current is the ac load current. The other two switching actions, one turn-on and one turn-off during the shoot-through interval, are imposed by the unique shoot-through actions. The switched voltage is still the peak dc-link voltage while the switched current is $2 / 3 I_{L}+i_{a} / 2$. Thus, the energy loss, $w_{1}$, within switching cycle can be approximated using (10) and
(11), as given by (26).

$$
\begin{equation*}
w_{1}=w_{\text {Tonoff }}\left(V_{p n},\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right)\right)+w_{\text {Tonoff }}\left(V_{p n}, i_{a}\right) \tag{26}
\end{equation*}
$$

where $w_{1}$ is the energy loss of $T_{p a}$ in a switching cycle when $i_{a}$ is positive. $w_{\text {Tonoff }}$ has the same form of $w_{\text {off }}(u, i)$ or $w_{\text {on }}(u, i)$ where the coefficients $K_{\text {Toni }}$ or $K_{\text {Toffi }}$ becomes $K_{\text {Tonoffi }}=K_{\text {Toni }}+K_{\text {Toffi, }} i=1,2, . .5 ; V_{p n}$ is the peak dc-link voltage of $Z$-source inverter; $I_{L}$ is in (19) ; $i_{a}$ is the load current.

When ac load current is negative, i.e. $i_{a}<0$, the $T_{p a}$ carries the current in the same way as that of $T_{n a}$ with $i_{a}>0$ in. Thus, switching actions of $T_{p a}$ can be illustrated by $i_{\text {Tna }}$ of $T_{n a}$ in Fig. 4. The waveform of $i_{n a}$ shows that the most left-hand turn-off is zero-voltage switching actions because the terminal voltage of $T_{n a}$ is clamped to almost zero voltage by the anti-parallel diode $D_{n a}$, which matches the most right-hand turn-on. Hence, the relevant switching losses may be ignored. Only turn-on/off occurring in the middle position is taken into account. From the approximation model in (10) and (11), one has

$$
\begin{equation*}
w_{2}=w_{\text {Tonoff }}\left(V_{\text {pn }},\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right)\right) \tag{27}
\end{equation*}
$$

where $w_{2}$ is the switching energy loss of $T_{p a}$ per PWM cycle in the negative-half cycle; $w_{\text {Tonoff }}$ was defined in (26). Thus

$$
\begin{align*}
P_{\text {Tpa,sw}} & =f_{s w}\left[\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{3 \pi}{2}+\Phi}\left(w_{1}+w_{2}\right) d \varphi\right] \\
& =f_{s w}\left[\begin{array}{l}
\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{3 \pi}{2}+\Phi} w_{\text {Tonoff }}\left(V_{p n},\left(\frac{2}{3} I_{L}+\frac{i_{a}}{2}\right) d \varphi\right. \\
+\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi} w_{\text {Tonoff }}\left(V_{p n}, i_{a}\right) d \varphi
\end{array}\right] \tag{28}
\end{align*}
$$

where $P_{T p a, s w}$ is the average switching loss in one line cycle. Replacing $i_{a}$ by that in (1) and (2) with some trigonometric operation,
$P_{T p a, s w}=f_{s w}\left[\begin{array}{l}\frac{3}{2} K_{T 3} V_{p n}{ }^{2}+\left(K_{T 1} V_{p n}+K_{T 4} V_{p n}{ }^{2}\right)\left(\frac{3}{2 \pi} I_{p k}+\frac{2}{3} I_{L}\right) \\ +\left(K_{T 2} V_{p n}+K_{T 5} V_{p n}{ }^{2}\right)\left(\frac{1}{\pi} I_{p k} I_{L}+\frac{5}{16} I_{p k}{ }^{2}+\frac{4}{9} I_{L}{ }^{2}\right)\end{array}\right]$
where $K_{T i}=K_{\text {Ton }, i}+K_{\text {Toff }, I}, \mathrm{i}=1,2, . .5 ; f_{\text {sw }}$ is the switching frequency; $V_{p n}, I_{L}$ and $I_{p k}$ were given by (5), (19) and (20) respectively.

### 4.2.2 Switching loss cause by Diode Dna in Zsource Inverter

$D_{n a}$ carries current only in the positive half cycle of load current. The turn-on switching loss of a diode is ignored. The waveform of $i_{\text {Dna }}$ in Fig. 4 shows that the most righthand turn-off of the diode is a zero-voltage turn-off due to the switched voltage clamped by $T_{n a}$, hence, the relevant switching loss is ignored. Thus, only the turn-off at the lefthand contributes to switching loss energy

$$
\begin{equation*}
w_{\text {Doff }}=w_{\text {Doff }}\left(V_{p n}, i_{a}\right) \tag{30}
\end{equation*}
$$

where $V_{p n}$ was given in (5); $i_{a}$ is the load current.
The average switching power loss of $D_{n a}$ in one linecycle is

$$
\begin{align*}
P_{D n a, s w} & =f_{s w}\left[\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{3 \pi}{2}+\Phi} w_{D o f f} d \varphi\right]=f_{s w}\left[\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi} w_{D o f f} d \varphi\right] \\
& =f_{s w}\left[\frac{1}{2 \pi} \int_{-\frac{\pi}{2}+\Phi}^{\frac{\pi}{2}+\Phi} w_{D o f f}\left(V_{p n}, i_{a}\right) d \varphi\right] \tag{31}
\end{align*}
$$

Replacing $i_{a}$ by the $I_{a}(t)$ in (1), the average switching power loss of the FWD can be approximated by (32).

$$
P_{D n a, s w}=f_{s w}\left[\begin{array}{c}
K_{D 3} V_{p n}{ }^{2}+\left(K_{D 1} V_{p n}+K_{D 4} V_{p n}{ }^{2}\right) \frac{1}{\pi} I_{p k}  \tag{32}\\
+\left(K_{D 2} V_{p n}+K_{D 5} V_{p n}{ }^{2}\right) \frac{1}{4} I_{p k}{ }^{2}
\end{array}\right]
$$

where $K_{D i}=K_{D o f f, i}, \quad i=1,2, . .5 ; \quad f_{s w}$ is the switching frequency; $V_{p n}$ and $I_{p k}$ were given by (5) and (20) respectively.

### 4.2.3 Total average switching loss in Z-source Inverter

Since the switching power loss is evenly distributed, the total average switching loss can be approximated by

$$
\begin{align*}
P_{2, s w} & =6 \times\left(P_{T p a, s w}+P_{D n a, s w}\right) \\
& =6 \times f_{s w}\left[\begin{array}{l}
\frac{3}{2} K_{T 3} V_{p n}{ }^{2}+\left(K_{T 1} V_{p n}+K_{T 4} V_{p n}{ }^{2}\right)\left(\frac{3}{2 \pi} I_{p k}+\frac{2}{3} I_{L}\right)+ \\
\left(K_{T 2} V_{p n}+K_{T 5} V_{p n}{ }^{2}\right)\left(\frac{1}{\pi} I_{p k} I_{L}+\frac{5}{16} I_{p k}{ }^{2}+\frac{4}{9} I_{L}{ }^{2}\right)+ \\
K_{D 3} V_{p n}{ }^{2}+\left(K_{D 1} V_{p n}+K_{D 4} V_{p n}{ }^{2}\right) \frac{1}{\pi} I_{p k}+ \\
\left(K_{D 2} V_{p n}+K_{D 5} V_{p n}{ }^{2}\right) \frac{1}{4} I_{p k}{ }^{2}
\end{array}\right] \tag{33}
\end{align*}
$$

where parameters have been defined in (29) And (32).

## 5. Average CASLs Model of the Threephase ac-dc Matrix Converter

### 5.1 Average conduction-loss model of ac-dc matrix converter

It is well known that ac load current dominates the average conduction loss of an ac-ac matrix converter. For the case of ac-dc matrix, dc load current dominates

$$
\begin{equation*}
P_{m c, c}=2\left[\left(V_{T}+V_{D}\right) I_{d c}+\left(r_{T}+r_{D}\right) I_{d c}{ }^{2}\right] \tag{34}
\end{equation*}
$$

where $P_{m c, c}$ is the average conduction loss; the square bracketed item is the conduction loss per output phase.

### 5.2 Average switching loss model of ac-dc matrix converter

The model of switching loss of ac-dc matrix converter is not as obvious as that of the conduction loss. For the convenience of discussion, Fig. 5 presents the timing diagram of a switching cycle in an arbitrary $60^{\circ}$-sector of line-cycle; Fig. 6 illustrates the detail switching actions within the commutation period $T_{C M}$ immediately after the instant $t_{C M p 1}$ in Fig. 5. The basic condition for the discussion is that the sequence of current commutation is in the way of " $a \rightarrow b \rightarrow c c \rightarrow b \rightarrow a$ " all the time as shown in Fig. 5, and that the four-step method in Fig. 6 is used for every commutation. The ac source is assumed three-phase balanced. Gate-drives for individual switches are assumed
ideal and lossless. Switched voltage that is equal to the forward voltage drop of IGBT or FWD does not contribute to switching loss.


Fig. 5 One instance of switching states of the circuit in Fig. 2 in one switching cycle $T$, where $x$ and $y$ are the intervals of active states; $z_{a}, z_{b}$, and $z_{c}$ are the ones of zero-states; $t_{C M p i}$ and $t_{C M n i}(i=1,2,3,4)$ are the instants of current commutation of phase $p$ and $n$ respectively

At instant $t_{C M P 1}$ in the left-hand half switching cycle of Fig. 5, when $i_{p}$ commutates from phase $a$ to $b$, the switching actions are the turn-off of the switch-cell $S_{p a}$ and the turn-on of $S_{p b}$; no switching action occurs in the switchcell $S_{p c}$; the switched voltage is $v_{a b}$. Examining Fig. 6. shows that, when $i_{p}$ is positive, i.e. the forward current, the turn-off of the backward switch $S_{p a B}$ at $t_{1}$ is a zero-current turn-off (ZC-off) not producing switching loss due to the absence of backward current; the turn-on of the forward switch $S_{p b F}$ at $t_{2}$ is a zero-current turn-on (ZC-on) not producing switching loss because its reversed terminal voltage under the condition of line voltages $\left(V_{a}(t)>V_{b}(t)\right)$ blocks the forward current during the turn-on; the turn-off of the forward switch $S_{p a F}$ at $t_{3}$ is a non-zero-current turnoff (NZC-off) certainly producing switching loss due to the existence of switched current; the turn-on of the backward switch $S_{p b B}$ at $t_{4}$ is a ZC -on not producing switching loss due to the absence of backward current. At instant $t_{\text {CMp } 4}$ with the same switched voltage of $v_{a b}$, by symmetry of the switching states as shown in Fig. 5, switch-cell $S_{p a}$ produces the switching loss through a non-zero-current
turn-on (NZC-on) of forward switch $S_{p a F}$ when the forward current $i_{p}$ commutates from phase $b$ back to $a$. Similar examination applied to the two instants at $t_{C M p 2}$ and $t_{C M p 3}$ shows that the NZC-on and NZC-off occur in the switchcell $S_{p c}$, and produce switching loss at the two instants respectively when forward current commutates between phase $b$ and $c$ with the switched voltage of $v_{b c}$. Details of the examination are ignored for abbreviation. Thus, with the common switched current, two combinations of NZCon and NZC-off produce switching loss with two switched voltages $v_{a b}$ and $v_{b c}$ respectively.

Note that the switching cycle and $60^{\circ}$-sector of linecycle are arbitrarily selected in the aforementioned examination, and that the line-to-line voltage $v_{c a}$ has no chance to be the switched voltage due to the definite absence of the current commutation between phase $a$ and $c$ for the given commutation sequence of " $a \rightarrow b \rightarrow c c \rightarrow b \rightarrow a$ " all the time. The examination may be applied to the other $60^{\circ}$-sectors, revealing that the difference in the switching actions between different sectors is merely the way how the two combinations of the NZC-on and NZC-off are allocated, as shown in Table 3. Considering the IGBTs and FWDs used in the switch-cells are identical, the common facts from the examination about the switching loss of output phase $p$ with forward current is that the switching loss is caused by two and only two combinations of NZCon and NZC-off at switched voltages $v_{a b}$ and $v_{b c}$ respectively.

Therefore, using (10), (11), and (12), switching energy within one switching cycle in the output phase $p$ may be approximated as given by (35).

$$
\begin{align*}
w_{s w} & =\left[\begin{array}{l}
w_{\text {Ton }}\left(v_{a b}, I_{d c}\right)+w_{\text {Toff }}\left(v_{a b}, I_{d c}\right)+w_{\text {Doff }}\left(v_{a b}, I_{d c}\right)+ \\
w_{\text {Ton }}\left(v_{b c}, I_{d c}\right)+w_{\text {Toff }}\left(v_{b c}, I_{d c}\right)+w_{\text {Doff }}\left(v_{b c}, I_{d c}\right)
\end{array}\right] \\
& =w_{\text {TonoffDoff }}\left(v_{a b}, I_{d c}\right)+w_{\text {TonoffDoff }}\left(v_{b c}, I_{d c}\right) \\
& =w_{s w}\left(K_{i}, v_{a b}, I_{d c}\right)+w_{s w}\left(K_{i}, v_{b c}, I_{d c}\right) \tag{35}
\end{align*}
$$

where $K_{i}=K_{i T o n}+K_{\text {iToff }}+K_{i D o f f} ; K_{i T o n}, K_{i T o f f}$, and $K_{i D o f f}$ are in Table 2( $i=1,2, . .5) ; w_{s w}\left(K_{i}, v_{a b}, I_{d c}\right)$ and $w_{s w}\left(K_{i}, v_{b c}\right.$, $I_{d c}$ ) are the switching energy loss when switched voltage are $v_{a b}$, and $\underline{v}_{b c}$ respectively; $I_{d c}$ is the forward current equal to the output dc current. From the model (6), $v_{a b}$ and $v_{b c}$ are

$$
\begin{equation*}
v_{a b}=\sqrt{3} V_{i n} \cos \left(\varphi_{i n}+\pi / 6\right) ; \quad V_{b c}=\sqrt{3} V_{i n} \cos \left(\varphi_{i n}-\pi / 2\right) \tag{36}
\end{equation*}
$$

where $\varphi_{i n}=\omega_{i n} t ; \omega_{i n}$ and $V_{i n}$ were defined in (6).


Fig. 6 Illustration of four-step commutation at the instant $t_{C M p 1}$ in Fig. 5 when the forward current of output phase $p$ commutates from phase $a$ to $b$ (a) the simplified circuit, where each physical switch is comprised of a combination of one IGBT and one FWD in series; $F$ and $B$ denote forward and backward respectively. (b) the timing diagram of four-step commutation, where $S_{p a}$ and $S_{p b}$ are logic signals for switch-cells; $S_{p j k}(j=a, b ; k=F$, $B$ ) are on/off states of the physical switches; $T_{C M}$ is the commutation period.

Table 3 Feature of switching actions of the switch-cells connected to the output phase $p$ in each $60^{\circ}$-sector of line-cycle according to the given sequence of current commutation and the four-step commutation method

|  | Sectors | Switchcells | $\mathrm{a} \rightarrow \mathrm{b}$ | $\mathrm{b} \rightarrow \mathrm{c}$ | $\mathrm{c} \rightarrow$ b | $\mathrm{b} \rightarrow \mathrm{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $v_{a}>v_{b}>v_{c}$ | $\begin{gathered} S_{p a} \\ S_{p b} \\ S_{p c} \end{gathered}$ | NZC- <br> off ZC-on | $\begin{aligned} & \text { NZC- } \\ & \text { off } \\ & \text { ZC-on } \end{aligned}$ | $\begin{aligned} & \text { NZC- } \\ & \text { on } \\ & \text { ZC-off } \end{aligned}$ | NZC- <br> on <br> ZC-off |
| 2 | $v_{b}>v_{c}>v_{a}$ | $\begin{aligned} & \hline S_{p a} \\ & S_{p b} \\ & S_{p c} \end{aligned}$ | $\begin{aligned} & \hline \text { ZC -off } \\ & \text { NZC - } \\ & \text { on } \end{aligned}$ | NZC- <br> off <br> ZC-on | NZC- <br> on ZC-off | ZC -on <br> NZC - <br> off |



From (35) and(36), by some trigonometric operation, the average switching loss in the output phase $p$ in one switching cycle, $p_{\text {swsw }}$, can be approximated as given by (37).

$$
\begin{align*}
p_{s w s w} & =f_{s w} w_{s w} \\
& =f_{s w}\left(w_{s w}\left(K_{i}, v_{a b}, I_{d c}\right)+w_{s w}\left(K_{i}, v_{b c}, I_{d c}\right)\right) \\
& =A \sqrt{3} V_{i n} \cos \left(\varphi_{i n}-\pi / 6\right)+3 B V_{i n}^{2}\left(1-0.5 \cos 2\left(\varphi_{i n}-\pi / 6\right)\right) \tag{37}
\end{align*}
$$

where $f_{s w}$ is the switching frequency; $A=K_{1} I_{d c}+K_{2} I_{d c}{ }^{2}$ and $B=K_{3}+K_{4} I_{d c}+K_{5} I_{d c}{ }^{2} ; K_{i}(i=1,2, \ldots 5)$ were defined in (35); $\varphi_{\text {in }}=\omega_{i n} t ; \omega_{\text {in }}$ and $V_{i n}$ were defined in (6).

The average switching loss in the output phase $p$ in one line-cycle, $p_{\text {swph }}$, is

$$
\begin{equation*}
p_{s w p h}=\frac{1}{2 \pi} \int_{0}^{2 \pi} p_{s w s w} d \varphi_{i n}=\frac{3}{\pi} \int_{-\pi / 6}^{\pi / 6} p_{s w s w} d \varphi_{i n}=f_{s w}\left(1.43 A V_{i n}+2.38 B V_{i n}{ }^{2}\right) \tag{38}
\end{equation*}
$$

where $A$ and $B$ were defined in (37); $V_{\text {in }}$ was defined in (6).
The average switching loss in output phase $n$ is the same as that in output phase $p$ because of the balanced input voltages and the identical switch-cells. Therefore, the average switching power loss of the whole ac-dc matrix converter is
$P_{m c, s w}=2 \times p_{\text {swph }}=f_{\text {sw }}\left[\begin{array}{l}4.76 K_{3} V_{\text {in }}{ }^{2}+\left(2.86 K_{1} V_{i n}+4.76 K_{4} V_{\text {in }}{ }^{2}\right) I_{d c} \\ +\left(2.86 K_{2} V_{\text {in }}+4.76 K_{5} V_{\text {in }}{ }^{2}\right) I_{d c}{ }^{2}\end{array}\right]$
where $K_{i}(i=1,2, \ldots .5)$ were defined in (35); $f_{s w}$ is the switching frequency; $I_{d c}$ and $V_{i n}$ were defined in (6). Thus, the average switching power loss, $P_{m c, s w}$, of the whole acdc matrix converter has dependency on output dc load current and input ac peak phase voltage. Considering the relation in (8), one has

$$
\begin{equation*}
V_{i n}=4 V_{d c} /(3 \sqrt{3} M) \tag{40}
\end{equation*}
$$

Replacing $V_{i n}$ in (39) with that in (40), the $P_{m c, s w}$ becomes

$$
P_{m c, s w}=f_{s w}\left[\begin{array}{l}
\frac{2.12 K_{3} V_{d c}{ }^{2}}{3 / 4 M^{2}}+I_{d c}\left(\frac{1.91 K_{1} V_{d c}}{\sqrt{3} / 2 M}+\frac{2.12 K_{4} V_{d c}{ }^{2}}{3 / 4 M^{2}}\right)  \tag{41}\\
+I_{d c}{ }^{2}\left(\frac{1.91 K_{2} V_{d c}}{\sqrt{3} / 2 M}+\frac{2.12 K_{5} V_{d c}{ }^{2}}{3 / 4 M^{2}}\right)
\end{array}\right]
$$

where $f_{s w}$ is the switching frequency; $I_{d c}$ and $V_{d c}$ were defined in (6), and $M$ in (8).

It can be seen from (34) and (41) that the conduction and switching losses in an ac-dc matrix converter has no dependency on $\cos (\Phi)$, i.e. the power factor $(\mathrm{pf})$.

## 6. Average CASLs Model of the Proposed MZC

Thus far, the power loss approximation has been investigated for power-switch network in a Z-source inverter or ac-dc matrix converter. MZC marries up both structures of Z-source inverter and ac-dc matrix converter. Therefore, its power loss approximation for the powerswitch network can be examined now. Table 4 shows the conditions for the calculation. The calculated results will be converted into the form of "mWatts per Watt" as given by (42) for the convenience of visualization.

$$
\begin{equation*}
P_{\text {loss }}^{\prime}=\frac{P_{\text {loss }}}{P_{o}} \times 1000 \tag{42}
\end{equation*}
$$

where $P^{\prime}{ }_{\text {loss }}$ is the result to be visualized; $P_{\text {loss }}$ is the calculated loss at certain operating point indicated by a pair of modulation index $(M)$ and power factor $(\mathrm{pf})$ in Table; $P_{o}$ is defined in Table 4.

Table 4 Conditions for the calculation of power loss using the analytical model of conduction and switching losses of MZC

| Symbol | Values |  |  |
| :---: | :---: | :---: | :---: |
|  |  | For dc-ac <br> inversion | For ac-dc <br> rectification |
| $\mathbf{p f}$ | $\cos (\Phi)$ | $0.7<\mathrm{pf}<1.0$ | $0.7<\mathrm{pf}<0.9$ |
| $\boldsymbol{M}$ | Modulation index | $0.866<M<$ |  |
|  |  | 1.155 | $0.5<M<1.155$ |
| $\boldsymbol{V}_{\boldsymbol{d} \boldsymbol{c}}$ | dc voltage | 36 V | 42 V |
| $\boldsymbol{P}_{\boldsymbol{o}}$ | Active power | 2000 W | 2000 W |
| $\boldsymbol{f}_{\boldsymbol{s w}}$ | Switching <br> frequency | 10 kHz | 10 Hz |
|  |  |  |  |

### 6.1 Average CASLs Model of the proposed MZC in dc-ac inversion mode

### 6.1.1 Average conduction loss of the proposed MZC in dc-ac inversion mode

In dc-ac operation mode, the MZC operates as a Z-source inverter in which each switch consists of two commonemitter connected IGBTs with respective anti-paralleled FWDs. One of the two IGBTs is set at on-state all the time, and the other one serves as an active IGBT as shown in Fig. 7 (a) ${ }^{[1]}$. Fig. 7(b) presents the equivalent configuration. Fig. 7 (c) shows the equivalent model during the on-state of the active IGBT. For this reason, the $V_{T}, V_{D}, r_{T}$, and $r_{D}$, used in (18)-(25) for the approximation of average conduction loss are replaced by $V_{T+D}, V_{D+T}, r_{T+D}$, and $r_{D+T}$ respectively. $V_{T+D}, V_{D+T}, r_{T+D}$, and $r_{D+T}$ are given by (43) and (44).

$$
\begin{align*}
& V_{T+D}=V_{D+T}=V_{T}+V_{D}  \tag{43}\\
& r_{T+D}=r_{D+T}=r_{T}+r_{D} \tag{44}
\end{align*}
$$



Fig. 7 Equivalent model of IGBT and FWD in MZC when MZC is in dc-ac inversion mode, (a) illustrative gatedrive signals and common-emitter IGBT-pair, (b) active switching IGBT and the model of constant-on IGBT, (c) model of whole common-emitter IGBT-pair

Equations (18) and (24) share the same parameters,
hence the average conduction loss of $Z$-source inverter given by (25) can be arranged in (45)
$P_{m z-d c a c, c}=6 \times\left[\begin{array}{l}r_{T+D} I_{p k}^{2}\left(\frac{2-D_{0}}{8}\right)+r_{T+D}\left(\frac{2}{3} I_{L}\right)^{2} D_{0}+ \\ V_{T+D} I_{p k}\left(\frac{2-D_{0}}{2 \pi}\right)+V_{T+D} I_{L} D_{0}\end{array}\right]$

Where

$$
I_{L}=\frac{P_{o}}{V_{d c}} ; \quad I_{p k}=\frac{4(\sqrt{3} M-1) P_{o}}{3 M V_{d c} \cos \Phi} ; \quad D_{0}=1-\frac{\sqrt{3} M}{2}
$$

The calculated result using the given values for dc-ac inversion in Table 4 is visualized in Fig. 8(a1).

### 6.1.2 Average switching loss of the proposed MZC in dc-ac inversion mode

MZC in dc-ac inversion mode operates as the $Z$-source inverter. The average switching loss then can be approximated by (29), (32), and (33). One has Switching loss caused by IGBT $T_{p a}$

$$
P_{n c c-d a c c T p, s w}=f_{s w}\left[\begin{array}{l}
\frac{3}{2} K_{T 3} V_{p n}{ }^{2}+\left(K_{T 1} V_{p n}+K_{T 4} V_{p n}{ }^{2}\right)\left(\frac{3}{2 \pi} I_{p k}+\frac{2}{3} I_{L}\right) \\
+\left(K_{T 2} V_{p n}+K_{T 5} V_{p n}{ }^{2}\right)\left(\frac{1}{\pi} I_{p k} I_{L}+\frac{5}{16} I_{p k}{ }^{2}+\frac{4}{9} I_{L}{ }^{2}\right)
\end{array}\right]
$$

where $K_{T i}(i=1,2, .$.$) is no longer the same one in (29) the$ one in the following expression because of the extra diode in series with $T_{p a}$, shown in Fig.7.

$$
\begin{equation*}
K_{T i}=K_{\text {Ton }, i}+K_{\text {Toff }, i}+K_{\text {Doff }, i} \tag{47}
\end{equation*}
$$

Switching loss caused by FWD $D_{n a}$ is

$$
P_{m z c-d c a c D n a, s w}=f_{s w}\left[\begin{array}{c}
K_{D 3} V_{p n}{ }^{2}+\left(K_{D 1} V_{p n}+K_{D 4} V_{p n}{ }^{2}\right) \frac{1}{\pi} I_{p k}  \tag{48}\\
+\left(K_{D 2} V_{p n}+K_{D 5} V_{p n}{ }^{2}\right) \frac{1}{4} I_{p k}{ }^{2}
\end{array}\right]
$$

where $K_{D i}=K_{D o f f, j}(i=1,2, .$.$) is the same as defined (32),$ since the constant on-state IGBT in series with the diode is
merely offering a conduction path without active switching actions. The model for the approximation of the average switching loss of MZC in dc-ac inversion is

$$
\begin{align*}
P_{\text {nve-dacc,sw }} & =6 \times\left(P_{\text {mec-dacTpo,sw }}+P_{\text {mec-dacocma,sw }}\right) \\
& =6 \times f_{s w}\left\{\begin{array}{l}
{\left[\begin{array}{l}
\frac{3}{2} K_{T 3} V_{p n}{ }^{2}+\left(K_{T 1} V_{p n}+K_{T 4} V_{p n}{ }^{2}\right)\left(\frac{3}{2 \pi} I_{p k}+\frac{2}{3} I_{L}\right) \\
+\left(K_{T 2} V_{p n}+K_{T S} V_{p n}{ }^{2}\right)\left(\frac{1}{\pi} I_{p k} I_{L}+\frac{5}{16} I_{p k}{ }^{2}+\frac{4}{9} I_{L}{ }^{2}\right)
\end{array}\right]} \\
+\left[\begin{array}{c}
K_{D S} V_{p n}{ }^{2}+\left(K_{D 1} V_{p n}+K_{D 4} V_{p n}{ }^{2}\right) \frac{1}{\pi} I_{p k} \\
+\left(K_{D 2} V_{p n}+K_{D S} V_{p n}{ }^{2}\right) \frac{1}{4} I_{p k}{ }^{2}
\end{array}\right]
\end{array}\right\} \tag{49}
\end{align*}
$$

where

$$
I_{L}=\frac{P_{o}}{V_{d c}} ; \quad I_{p k}=\frac{4}{3} \frac{\sqrt{3} M-1}{M \cos \Phi} P_{o} ; \quad V_{p n}=\frac{1}{\sqrt{3} M-1} V_{d c}
$$

The calculated result using the given values for dc-ac inversion in Table 4 is visualized in Fig. 8(a2).

### 6.1.3 Average CASLs of MZC in dc-ac mode

By the results in the previous two sub-sections, the total average loss caused by power semiconductors in dc-ac mode of MZC can be approximated as given by (50).

$$
\begin{equation*}
P_{m \mathrm{wc}-\mathrm{dax}}=P_{m \mathrm{wc}-\mathrm{dax}, c}+P_{m \mathrm{mc}-\mathrm{dac}, \text { sw }} \tag{50}
\end{equation*}
$$

where $P_{m z-d c a c, c}$ and $P_{m z c-d c a c, s w}$ are given in (45) and (49) respectively. The calculated result using the given values for dc-ac inversion in Table 4 is visualized in Fig. 8(a3).

### 6.2 Average CASLs Model of the proposed MZC in ac-dc rectification mode

In ac-dc rectification, the MZC is operated as ac-dc matrix converter shown in Fig. 2. The conduction loss and switching loss models of ac-dc matrix converter in (34) and (41) of section 5 can be used straightaway. The conduction loss $P_{m z c-a c d c, c}$, switching loss $P_{m z c-a c d c, s w}$, and CASLs $P_{m z c-}$ acdc of MZC in ac-dc rectification mode are given by(51), (52), and (53) respectively.

$$
\begin{align*}
P_{m z c-a c d c, c} & =P_{m c, c} \\
& =2\left[\left(V_{T}+V_{D}\right) I_{d c}+\left(r_{T}+r_{D}\right) I_{d c}^{2}\right] \tag{51}
\end{align*}
$$

$$
\begin{align*}
P_{m z c-a c d c, s w} & =P_{m c, s w} \\
& =f_{s w}\left[\begin{array}{c}
\frac{2.12 K_{3} V_{d c}{ }^{2}}{3 / 4 M^{2}}+I_{d c}\left(\frac{1.91 K_{1} V_{d c}}{\sqrt{3} / 2 M}+\frac{2.12 K_{4} V_{d c}{ }^{2}}{3 / 4 M^{2}}\right) \\
+I_{d c}{ }^{2}\left(\frac{1.91 K_{2} V_{d c}}{\sqrt{3} / 2 M}+\frac{2.12 K_{5} V_{d c}{ }^{2}}{3 / 4 M^{2}}\right)
\end{array}\right]  \tag{52}\\
P_{m z c-a d c}= & P_{m z c-a d d c, c}+P_{m z c-a c d c, s w} \tag{53}
\end{align*}
$$

where $V_{d c}$ and $I_{d c}$ are dc voltage and dc current respectively; $M$ is modulation index, $0<M<2 / \sqrt{ } 3$; $K_{i}=K_{\text {Ton }, i}+K_{\text {Toff }, i}+K_{\text {Doff }, i}$ is the switching loss parameters given by Table $2 ; r_{T}, r_{D}, v_{T}$, and $v_{D}$ are On-state parameters given by Table 1.

Fig. 8(b1-b3) are the visualized results of conduction loss, switching loss, and CASLs of MZC in the condition of ac-dc rectification mode in Table 4, in which dc voltage is 42 V .

## 7. Conclusion

The analytical models for the approximation of the conduction and switching losses of the power-switch network in the MZC have been achieved.

When the MZC is in dc-ac inversion mode, the
analytical model for MZC is the same as that for the Zsource inverter except that the parameters of on-state resistance and forward voltage drop used in the approximation of conduction loss for MZC are higher than those for the $Z$-source inverter.

When the MZC is in ac-dc rectification mode, both the analytical models and the parameters in calculation are the same as that for the ac-dc matrix converter.

For the practice of engineering design work, the models developed in this paper are ready for use in the estimation of CASLs of Z-source inverters, ac-dc matrix converters, or MZC, after the acquisition of the experimental data of the on-state and switching energy of employed power-switches.

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Fig. 8 Visualized losses of MZC. (a1), (a2), and (a3) are conduction loss, switching loss, and CASLs of MZC respectively when $V_{d c}=36 \mathrm{~V}$ in dc-ac inversion mode; (b1), (b2), and (b3) are conduction loss, switching loss, and CASLs of MZC respectively when $V_{d c}=42 \mathrm{~V}$ in ac-dc rectification mode.
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