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Improvement in Control Performance of a Servo System Compensating Bandwidth Variations at Low Speed

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Abstract

This paper presents a novel design method for determining the PID gains of a speed controller for a servo system compensating variations in bandwidth at a low speed. The variations in bandwidth of a speed controller are measured at a low speed and the relationship between the bandwidth and the damping ratio are verified by determining the location of the closed loop pole. The proposed algorithm uses the z-transform of a plant and speed controller and applies the time-varying sampling method for determining the PID gains of the speed controller at low speed. The magnitude and the phase condition are considered for finding a suitable control gain. The usefulness and effectiveness of the proposed method is demonstrated through experimental results such as low speed control and robust disturbance responses.

Key Words: Bandwidth variation, Low speed, PID gains

I. Introduction

A PI (Proportional and Integral) controller is generally used as a method for speed control of motor drives in industrial servo systems. The P and I gains are determined by various factors such as the motor inertia, the torque constant and the bandwidth of the speed controller.

If the motor drive uses an incremental encoder as a position sensor and the rotor speed is calculated by the M/T method, the time delay in the speed control loop increases and the bandwidth of the speed controller decreases when the motor is controlled at a very low speed. This means the encoder pulse is detected less than 1 time per sampling period. The constant control gains in the low speed region decreases the control performance.

Conventional studies on the control of motors at low speed use an estimation of speed [1],[2], load torque and inertia [3] or a Kalman filter [4]. These methods focus on an additional algorithm for the improvement of control performance at low speed. However, these algorithms don't consider the performance improvement of the speed controller itself at low speed.

This paper proposes a novel method for determining the PID gains according to the variations in speed control bandwidth at low speed. The variations in speed control bandwidth at low speed are measured and the relation between the bandwidth and the damping ratio is verified by determining the location of the closed pole.

The proposed algorithm uses a z-transform of the motor drive and applies the time-varying sampling method. The proper PID gains at low speed are determined by applying the magnitude and the phase condition in the z-domain. The experimental results verify the improved response and robustness of the proposed method at low speed.

II. PID Tuning in Low Speed Compensation Variation of Control Bandwidth

A. Analysis of the time delay at low speed

The M/T method is one of the most commonly used algorithms for speed measurement of servo motors [5]. The speed is calculated by counting the pulses of the incremental encoder and the internal timer of the microprocessor.

However, if the servo motor is controlled at a very low speed, no pulses from the encoder are injected to the controller during one sampling period. In this case, the time of the speed measurement is delayed and it decreases the bandwidth of the speed controller.

For example, assume that a 3600 pulse incremental encoder is used for a servo drive system and the sampling period of speed controller is $1[msec]$. If the servo motor is controlled at less than $16[rpm]$, no pulses are generated from the encoder during 1 cycle of the speed control.

In this case, the time delay of the speed measurement can be calculated using the internal counter of the microprocessor.

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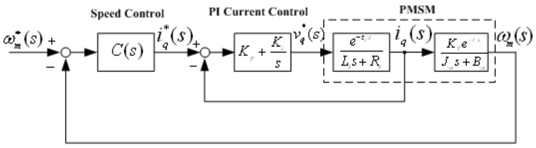

Fig. 1. Block diagram of the standard speed controller in s-domain.

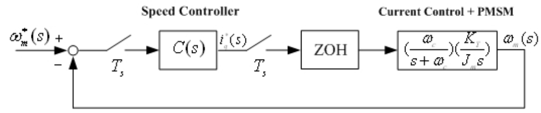

Fig. 2. Block diagram of approximated speed control loop of motor in z-domain.

The time delay of the speed measurement at low speed is shown in Table I.

TABLE I
TIME DELAY OF THE SPEED MEASUREMENT IN LOW SPEED

Speed[rpm]	Time delay[μsec]
14	205
13	290
12	405
11	530
10	680
9	875
8	1115
7	1430
6	1850
5	2450
4	3200

B. Mathematical expression of the speed control loop of motor drives

Fig. 1 shows the block diagram of a standard speed controller and a PMSM (Permanent Magnet Synchronous Motor) in the s-domain [6]. The current controller generates the reference voltage of the motor and the speed controller outputs the current reference $i_q^*(s)$. The current controller can be simplified as a time delay function because the bandwidth of the current controller is much higher than the speed controller. K_p and K_i are the gains of the current controller. τ_c, L_s and R are the time delay of current control loop, the inductance and the stator resistance, respectively. J_m, B_m and τs are the motor inertia, the friction and the time delay of the speed control loop, respectively.

The open-loop transfer function of the speed control loop in the s-domain is shown in (1).

$$G_p(s) = \left(\frac{\omega_c}{s+\omega_c}\right)\left(\frac{K_T}{J_{m^s}+B_m}\right) \tag{1}$$

where, τ_c is the bandwidth of the current controller and K_T is the torque constant.

Fig. 2 shows the block diagram of a standard speed control loop in the z-domain. The open-loop transfer function of the speed control loop in the z-domain can be expressed as shown in (2).

$$\begin{aligned} G_p(z) &= z\left[\left(\frac{1-e^{-Ts}}{s}\right)G_p(s)\right] = (1-z^{-1})z\left(\frac{G_p(s)}{s}\right) \\ &= \frac{\left(\frac{bT}{a}-\frac{b}{a^2}+\frac{bT}{a^2}e^{-aT}\right)z+\left(\frac{b}{a}\right)\left(\frac{1}{a}-\frac{e^{-aT}}{a}-Te^{-aT}\right)}{z^2-(1+e^{-aT})z+e^{-aT}} \quad (2) \\ &\left(a=\omega_c, b=\frac{K_{T\omega_c}}{J_m}\right) \end{aligned}$$

where, T is the sampling time of the speed controller.

Using the M/T method at low speed, the sampling period of the speed controller is varied according to the motor speed.

$$\begin{aligned} G_p(z) &= z\left[\left(\frac{1-e^{-Ts}}{s}\right)G_p(s)\right] \quad (3) \\ &= (1-z^{-1})z\left(\frac{G_p(s)}{s}\right) \\ &= \frac{\left(\frac{bT}{a}-\frac{b}{a^2}+\frac{bT}{a^2}e^{-aT}\right)z+\left(\frac{b}{a}\right)\left(\frac{1}{a}-\frac{e^{-aT}}{a}-Te^{-aT}\right)}{z^2-(1+e^{-aT})z+e^{-aT}}. \end{aligned}$$

The PID controller can be expressed in the s-domain as shown in (4) and in the z-domain as shown in (5).

$$G_c(z) = K_{sp} + K_{sd^s} + \frac{K_{si}}{s} \tag{4}$$

$$\begin{aligned} G_c(z) &= K_{sp} + \frac{K_{sd}(z-1)}{Tz} + \frac{K_{si}T(z+1)}{2(z-1)} \quad (5) \\ &= \frac{(K_{si}T^2+sK_{sp}T+2K_{sd})z^2+(K_{si}T^2-2K_{sp}T-4K_{sd})z+2K_{sd}}{sTz(z-1)} \end{aligned}$$

The transfer function of the open loop control system is expressed as (6).

$$G_0(z) = G_p(z) \times G_c(z). \tag{6}$$

After finding the transfer function, the location of the closed loop pole has to be determined. The closed loop pole in the s-domain is expressed as (7).

$$\begin{aligned} &s = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -\delta\omega_n + j\omega_d \quad (7) \\ &(\omega_d = \omega_n\sqrt{1-\delta^2}) \end{aligned}$$

where, ξ is the damping ratio, ω_n is the natural frequency and ω_d is the damping frequency of oscillation.

In the z-domain, (7) is transformed as (8).

$$\begin{aligned} z &= e^{Ts} = exp(-\zeta\omega_n T + j\omega_d T) \\ &= exp\left(-\frac{2\pi\delta}{\sqrt{1-\delta^2}}\frac{\omega_d}{\omega_s} + j2\pi\frac{\omega_d}{\omega_s}\right) \ (T = \frac{2\pi}{\omega_s}). \quad (8) \end{aligned}$$

Therefore, the location of the closed loop pole in the z-domain can be expressed as shown in (9).

$$|z| = exp(-\frac{2\pi}{\sqrt{1-\delta^2}}\frac{\omega_d}{\omega_s}), \angle z = 2\pi\frac{\omega_d}{\omega_s}. \tag{9}$$

For determining ω_d/ω_s, the natural frequency ω_n has to be found. The natural frequency ω_n is selected to be 0.923[rad/sec] in this paper, because the PID gains are too large to control the servo motor if ω_n is larger than 0.923.

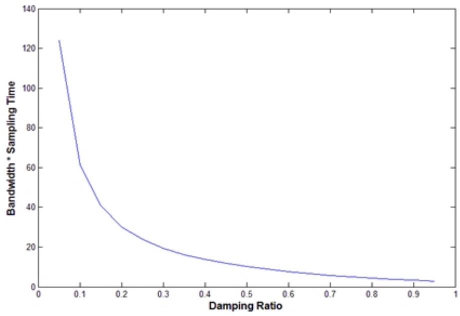

Fig. 3. The relation between $\omega_{BW} \times T_s$ and damping ratio ξ.

The bandwidth of the speed controller can be derived as the following.

The closed loop transfer function of the speed controller is shown in (10).

$$\frac{G_0(s)}{1+G_0(s)} = \frac{\left(k_{ps} + \frac{k_{is}}{s} + k_{ds^s}\right)\left(\frac{\omega_c}{s+\omega_c}\right)\left(\frac{K_T}{J_{m^s}}\right)}{1+\left(k_{ps} + \frac{k_{is}}{s} + k_{ds^s}\right)\left(\frac{\omega_c}{s+\omega_c}\right)\left(\frac{K_T}{J_{m^s}}\right)} \tag{10}$$

By simplifying (10) as a second order transfer function, it can be expressed as (11).

$$\frac{G_0(s)}{1+G_0(s)} = \frac{\omega_n^2}{s^2+\omega_n\zeta s+\omega_n^2}. \tag{11}$$

Therefore, the amplitude of (11) at ω_{BW} is expressed as shown in (12).

$$\left|\frac{\omega_n^2}{s^2+\omega_n\delta s+\omega_n^2}\right|_{\omega=\omega_{BW}} = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega_{BW})^2+4\delta^2\omega_n^2\omega_{BW}^2}} = \frac{1}{\sqrt{2}} \tag{12}$$

where, ω_{BW} is the bandwidth of the speed controller.

Therefore, the bandwidth of the speed controller can be calculated as (13),

$$\omega_{BW} = \omega_n\sqrt{(1-2\delta^2)+\sqrt{4\delta^4-4\delta^2+2}} \tag{13}$$

and using the relation of (14),

$$\omega_n = 4/T_s\delta. \tag{14}$$

The relationship between the value of the bandwidth and the damping ratio can be expressed as (15).

$$\omega_{BW} \times T_s = \frac{4}{\delta}\sqrt{(1-2\delta^2)+\sqrt{4\delta^4-4\delta^2+2}}. \tag{15}$$

If the sampling time is constant, the bandwidth of the speed controller decreases when the damping ratio grows as shown in Fig. 3. Therefore, if the motor is controlled at low speed, the bandwidth of the speed controller decreases and the damping ratio should be increased.

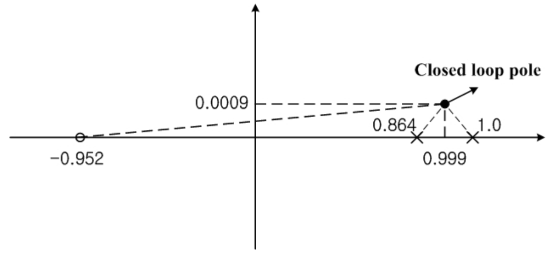

Fig. 4. The pole-zero plot of z-domain open-loop transfer function.

The bandwidth of the speed controller is determined by injecting a sinusoidal speed reference and increasing its frequency. When the phase of the speed response of the PMSM is delayed -45 degree, that point of frequency is the bandwidth of the speed controller. From (9) and (15), the location of the closed loop pole and the damping ratio can be determined as shown in Table II.

TABLE II
BANDWIDTH, DAMPING RATIO AND ω_d/ω_s IN LOW SPEED

Speed[rpm]	Bandwidth [rad/sec]	Damping ratio	ω_d/ω_s	Closed loop pole
14	122.33	0.568	1/6892.18	0.9994+i0.00089
13	113.47	0.613	1/6678.38	0.9993+i0.00094
12	104.55	0.642	1/6131.73	0.9991+i0.00100
11	101.52	0.653	1/5874.09	0.9991+i0.00110
10	98.49	0.663	1/5412.05	0.9990+i0.00120
9	95.46	0.675	1/4917.15	0.9988+i0.00130
8	92.43	0.686	1/4423.12	0.9987+i0.00140
7	83.21	0.723	1/4054.56	0.9984+i0.00150
6	73.98	0.765	1/3708.35	0.9980+i0.00170
5	64.76	0.813	1/3384.34	0.9974+i0.00190
4	55.54	0.868	1/3236.68	0.9966+i0.00190

Assume, $T_s = 0.0012[sec], K_T = 0.1132[Nm/A]$ and $J_m = 0.0101[kgm^2]$. From (6), the pole-zero plot of the z-domain open-loop transfer function is shown as in Fig. 4.

For determining the PID gains of a speed controller with a closed loop pole, two conditions should be considered. First, the phase of the closed loop pole should be -180 degrees.

From Fig. 4, the shortage angle from the view point of the plant and PID controller can be determined as (16).

$$-0.0514^\circ - 126.8956^\circ - 126.8956^\circ - 0.3783^\circ + 0.0263^\circ + 180^\circ = -74.195^\circ. \tag{16}$$

This value should be compensated by two zeros of the PID controller. The shortage angle which should be compensated by each zero is $-74.195^\circ \div 2 = -37.097^\circ$. The location of the zero of the PID controller can be determined to 0.9981 and the numerator of the transfer function of the PID controller is expressed as (17).

$$(z-0.9981)(z-0.9981) = z^2 - 1.9962z + 0.9962. \tag{17}$$

Two polynomial expressions are obtained from (17).

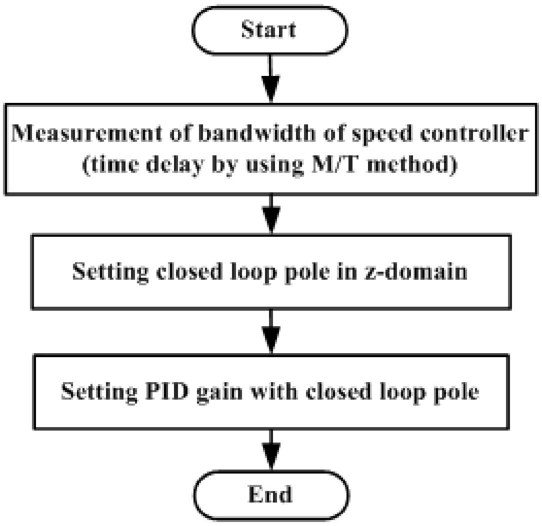

Fig. 5. Flowchart of the proposed algorithm.

$$\frac{K_{si}T^2 - 2K_{sp}T - 4K_{sd}}{K_{si}T^2 + 2K_{sp}T + 2K_{sd}} = -1.9962 \tag{18}$$

$$\frac{2K_{sd}}{K_{si}T^2 + 2K_{sp}T + 2K_{sd}} = 0.9962\ . \tag{19}$$

Second, by applying the magnitude condition that the magnitude of the closed loop pole has to be 1, the following polynomial is obtained.

$$(K_{si}T^2+2K_{sp}T+2K_{sd}) \left|\frac{(z-0.9981)^2}{2Tz(z-1)}\right| |G_p(z)|_{z=0.999+j0.0009}=1. \tag{20}$$

The PID gains are determined from (18), (19) and (20) as shown in Table III.

TABLE III
CALCULATED VALUE OF PID GAINS IN LOW SPEED

Speed[rpm]	P	I	D
14	0.1563	0.1217	0.0501
13	0.1616	0.1216	0.0536
12	0.1852	0.1372	0.0625
11	0.1837	0.1324	0.0662
10	0.1946	0.1355	0.0698
9	0.2036	0.1393	0.0743
8	0.2131	0.1433	0.0791
7	0.2490	0.1590	0.0974
6	0.3030	0.1828	0.1254
5	0.3925	0.2230	0.1724
4	0.5773	0.3071	0.2707

The flow of the proposed algorithm is shown in Fig. 5.

III. EXPERIMENTAL RESULTS

The microprocessor of the servo drive system for the experiments is a TMS320VC33 (Texas Instruments) and the configuration of the servo system is shown in Fig. 6 and 7.

Fig. 8 shows the overall block diagram of the servo drive system.

Fig. 6. Control board.

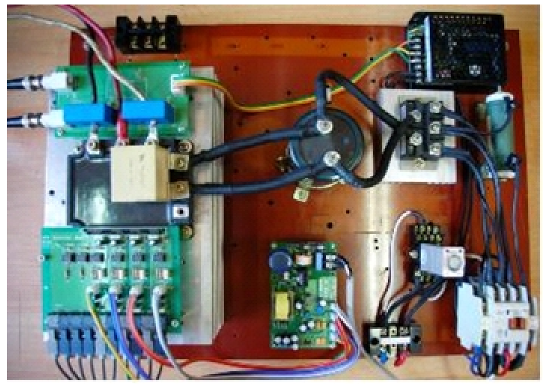

Fig. 7. Inverter board.

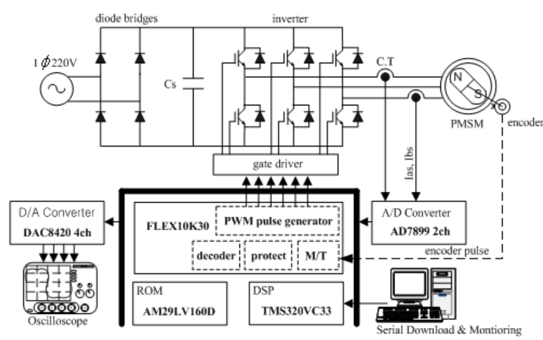

Fig. 8. Block diagram of the servo drive system.

The experiments are performed at low speed. The characteristics of the speed response and the disturbance response in the low speed range are shown in Fig. 9, 10, 11, and 12.

Fig. 9 and 10 show the speed response in the low speed range. Fig. 9 shows the speed response at a constant PID gains. If the speed reference is changed from 4[rpm] to 7[rpm] and the PID gains are constant, the waveform of its speed response contains ripple. On the other hand, as shown in Fig. 10, the speed response is improved and the speed ripple is reduced by compensating the bandwidth variation of the speed controller in the low speed region.

Fig. 11 shows the waveform of the disturbance response without compensation and Fig. 12 shows the disturbance response with the proposed algorithm in the low speed range. The servo motor is operating at 5[rpm] and the external

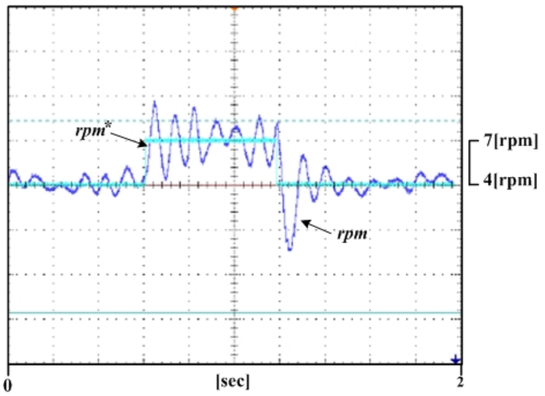

Fig. 9. Speed response of servo motor with constant PID gain.

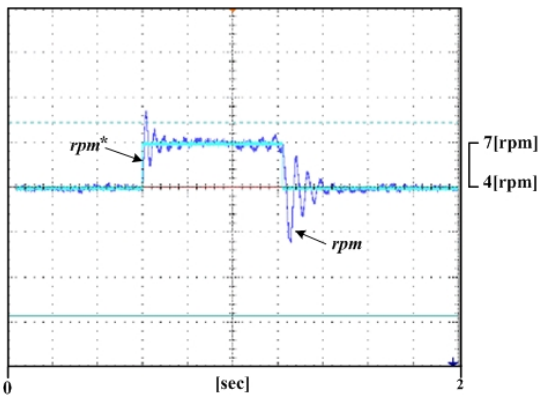

Fig. 10. Speed response of servo motor with compensating bandwidth variation.

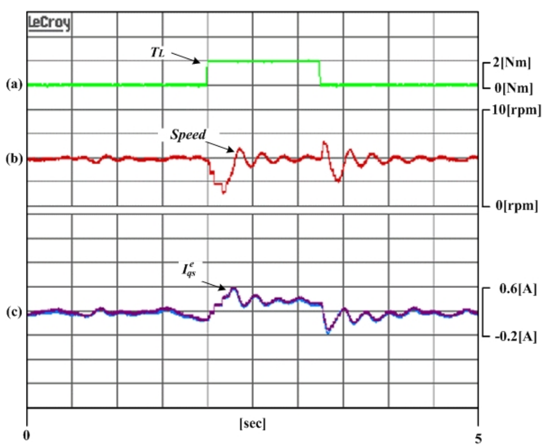

Fig. 11. Disturbance response when the PID gains are constant.
(a) Load torque of external PMSM
(b) Speed response of PMSM
(c)Q-axis current response of PMSM

disturbance is injected 2[Nm] during 2.5[sec] by the torque control of another coupled PMSM. The speed and q-axis current waveforms of Fig. 11 contain ripples in the transient state. Otherwise, the improved speed and q-axis current responses are obtained by implementing the proposed algorithm as shown in Fig. 12.

IV. Conclusions

his paper suggests a novel PID tuning method considering the variation of the bandwidth of a speed controller in the low speed range. The main contribution of this paper is the proof of the bandwidth variation of the PID gains in the low speed range. The experimental results verify the usefulness of the proposed algorithm.

The proposed algorithm can be easily implemented and requires less computation time.

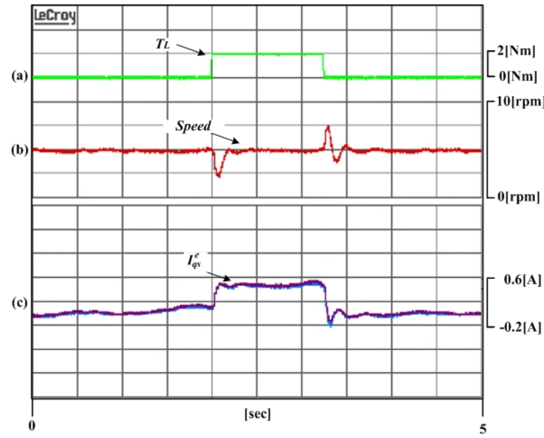

Fig. 12. Disturbance response with compensating bandwidth variation.
(a) Load torque of external PMSM
(b) Speed response of PMSM
(c) Q-axis current response of PMSM

Appendix

TABLE IV
PARAMETERS OF PMSM

Rated voltage	220[V]	Poles	8
Rated current	15.7[A]	Stator resistance	0.0746[Ω]
Rated speed	1800[rpm]	Stator inductance	1.144[mH]
Maximum speed	2000[rpm]	Back EMF constant	0.092[$V/rad/s$]

Acknowledgment

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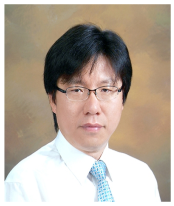

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