

# Elimination of Low Order Harmonics in Multilevel Inverters Using Genetic Algorithm

Reza Salehi\*, Naeem Farokhnia<sup>†</sup>, Mehrdad Abedi\*, and Seyed Hamid Fathi\*

<sup>†\*</sup> Dept. of Electrical Eng., Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran

## Abstract

The selective harmonic elimination pulse width modulation (SHEPWM) switching strategy has been applied to multilevel inverters to remove low order harmonics. Naturally, the related equations do not have feasible solutions for some operating points associated with the modulation index ( $M$ ). However, with these infeasible points, minimizing instead of eliminating harmonics is performed. Thus, harmful harmonics such as the 5<sup>th</sup> harmonic still remains in the output waveform. Therefore, it is proposed in this paper to ignore solving the equation associated with the highest order harmonics. A reduction in the eliminated harmonics results in an increase in the degrees of freedom. As a result, the lower order harmonics are eliminated in more operating points. A 9-level inverter is chosen as a case study. The genetic algorithm (GA) for optimization purposes is used. Simulation results verify the proposed method.

**Key Words:** Genetic Algorithm (GA), Multi-level Inverter, Selective Harmonic Elimination Pulse Width Modulation (SHEPWM)

## I. INTRODUCTION

In recent years, electrical industries have expanded and the variety of loads has increasingly grown. Recently, the industry has begun to apply high-voltage high-power equipment that has reached the megawatt range. Today, the direct connection of a single semiconductor switch to a system with medium sized voltage grids (2.3, 3.2, 4.16 and 6.9 KV) is too difficult [1]. To overcome the limitations on semiconductor voltage and current ratings, some type of series connection will be necessary. Therefore, multilevel inverters have been introduced. Due to their ability to synthesize waveforms with a better harmonic spectrum and attain a higher voltage without transformers, they have been receiving increasing attention in the past few years, especially in distributed generation systems (DG) [2], high voltage direct current transmissions (HVDC) [3], electrical drives [4] and flexible AC transmission systems (FACTS) devices [5].

Today, there are many switching strategies which are applied to multilevel inverter topologies. Some of these switching strategies are mentioned in [6]. Selective harmonic elimination pulse width modulation (SHEPWM) [6]–[8] is the most famous switching strategy. The aim of this method is the eliminating low order harmonics, if it is possible, or at least to minimize them [9]. The nature of these equations is nonlinear. Some iterative methods such as: the Newton-Raphson (N-R) have been used to solve the equations [10].

Iterative methods depend mainly on the initial guess and divergence problems are likely to occur. Resultant theory as an analytical method has been proposed in [11] to find all feasible solutions. However, this method is not applicable for a high number of levels. genetic algorithm (GA) has been proposed in [12]. The GA is simple and applicable to problems with any number of levels, without the extensive derivation of analytical expressions, for both eliminating and minimizing harmonics. This algorithm is chosen for the optimization goals.

In conventional SHEPWM, two sets of modulation indexes ( $M$ ) are specified. In the first set, the equations have feasible solutions, whereas in the second set there are not any analytical solutions. Therefore, when using these optimizing methods, the low order harmonics are minimized as much as possible but not removed completely. As a result, low order harmonics such as: the 5<sup>th</sup> harmonic still remain in the output voltage.

There are  $S$  switching angles associated with  $S$  cells. These angles are considered as variables or degrees of freedom. Thus,  $S$  equations can be satisfied.  $S-1$  equations are devoted to eliminating the ( $S-1$ ) low order harmonics starting from the 5<sup>th</sup> order for 3-phase applications or from the 3<sup>rd</sup> order for single phase applications. Also, one equation is dedicated to satisfying the fundamental component. The modulation index ( $M$ ) is defined as a symbol of the fundamental component. In some  $M$ s, which are focused on here, there are no feasible solutions, whereas some loads need to work in whole range of  $M$ s [13].

In this paper, a method to categorize all values of  $M$  in  $S$  groups is proposed. For each group, special equations are solved. In the first group, all  $S$  equations are satisfied and  $S-1$  harmonics are perfectly removed. In the second case,

Manuscript received Aug. 22, 2010; revised Jan. 30, 2011

<sup>†</sup> Corresponding Author: farokhnia@aut.ac.ir

Tel: +98-21-64543550, Amirkabir Univ. of Tech.

\*Dept. of Electrical Eng., Amirkabir University of Technology (Tehran Polytechnic), Iran

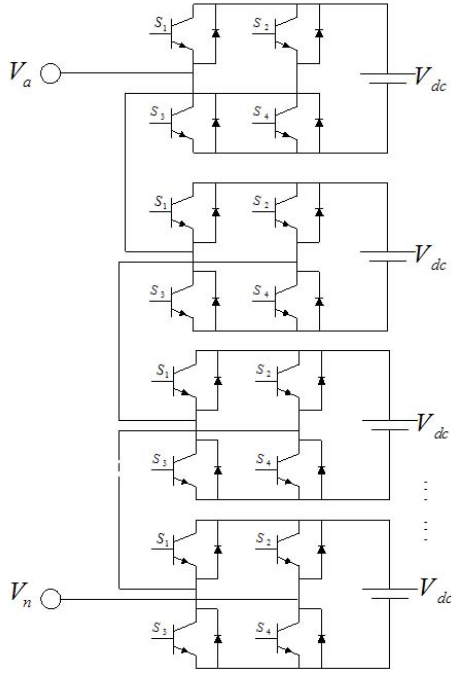


Fig. 1. Cascaded multilevel inverter.

the elimination of one equation is ignored. Thus, the new equations have more feasible solutions when compared with the first group. Those  $M_s$  which are not dependent on the first group are devoted to the second group. Subsequently, S-2 equations are solved and a third group is constructed. Finally, the last group is dedicated to satisfying only the fundamental component. These assumptions result in harmful lower order harmonics, such as 5<sup>th</sup> harmonic, being removed for more intervals of  $M_s$ .

Simulation based results confirm that the purposed method provides a suitable waveform with less harmonic pollution. A 9-level inverter is chosen as a case study.

## II. STRUCTURES OF MULTI-LEVEL INVERTERS

Three main topologies have been proposed for multilevel inverters. These topologies are diode clamped multi-level inverters (DCMI), flying-capacitor multi-level inverters (FCMI) and cascaded multi-level inverters with separate DC sources [14].

The cascaded multi-level inverter shown Fig. 1 has many advantages such as circuit layout flexibility, no extra clamping diodes or voltage balancing capacitors, an easily adjusted number of output voltage levels by adding or removing full-bridge cells, and the lowest number of components when compared with other inverters [15]. In addition, the switching devices turn on and turn off only one time per cycle to overcome the switching loss problem [16].

This multi-level inverter is made from several full-bridge inverters. The AC outputs for each different level of the full-bridge inverters are connected in series so that the synthesized voltage waveform becomes the sum of the inverter outputs. Fig. 2 shows the output waveform of a 9-level inverter.  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  are the switching angles of the multilevel inverter which are considered as variables and should be determined.

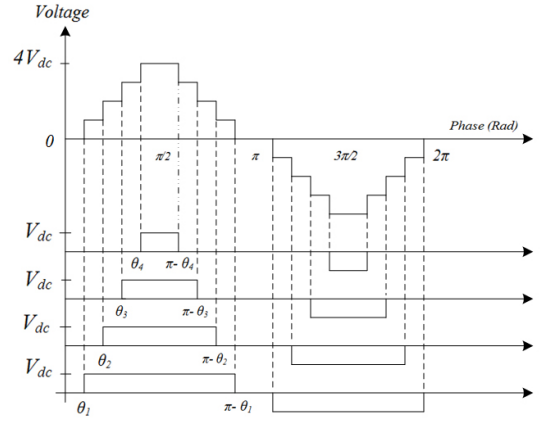


Fig. 2. Nine level waveform of cascaded multilevel inverter.

$V_{dc}$  is the voltage of the DC sources which are shown in Fig. 1. Here, it is supposed that the voltages of all of the DC sources are equal. Each full-bridge inverter produces a three level waveform  $+V_{dc}$ ,  $-V_{dc}$  and 0, and each angle  $\theta_i$  is related to inverter  $i$ , where,  $i = 1, 2, \dots, S$ .  $S$  is the number of DC sources that is equal to number of switching angles. The number of levels ( $L$ ) is calculated by  $L = 2S + 1$  [7].

## III. SWITCHING STRATEGIES

Nowadays, there are several switching strategies which are applied to multilevel inverter topologies such as: 1) sinusoidal pulse width modulation (SPWM); 2) selective harmonic elimination pulse width modulation (SHEPWM); 3) minimization of total harmonic distortion (MTHD). In MTHD, without any emphasis on special harmonics, all harmonics in the same weight (i.e. THD) are minimized [17], [18]. In SHEPWM, the low order harmonics are eliminated. There are some advantages and disadvantages to these three techniques which are reported in the literature [19]. In this paper, the SHEPWM technique is developed.

### A. Basic SHEPWM

Considering Fig. (2), with equal amplitude of all the DC sources, the Fourier series expansion of this waveform is:

$$V(\omega t) = \sum_{n=1}^{\infty} V_n \sin(n\omega t) \quad (1)$$

where,  $V_n$  is the amplitude of the harmonics. The angles are limited to between zero and  $90^\circ$  ( $0 \leq \theta \leq 90$ ). Because of an odd quarter-wave symmetric characteristic, the harmonics with an even order become zero. Subsequently,  $V_n$  becomes:

$$V_n = \begin{cases} \frac{4V_{dc}}{n\pi} \sum_{i=1}^S \cos(n\theta_i) & n : \text{odd} \\ 0 & n : \text{even} \end{cases} \quad (2)$$

In this paper, a 9-level inverter is chosen as a case study. Thus, with four angles as degrees of freedom, it is possible to satisfy the fundamental component and to eliminate three low order harmonics i.e. 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup>. It is not necessary to delete the triplen harmonics, because they will be eliminated

in the line to line output voltage in the  $Y$  connection, automatically. In other words, for a 9-level inverter, the following equation should be solved:

$$\begin{aligned} M &= [\cos(\theta_1) + \cos(\theta_2) + \dots + \cos(\theta_4)]/4 \\ 0 &= \cos(5\theta_1) + \cos(5\theta_2) + \dots + \cos(5\theta_4) \\ 0 &= \cos(7\theta_1) + \cos(7\theta_2) + \dots + \cos(7\theta_4) \\ 0 &= \cos(11\theta_1) + \cos(11\theta_2) + \dots + \cos(11\theta_4) \end{aligned} \quad (3)$$

where,  $M$  is Modulation Index and defined as:

$$M \triangleq \frac{V_1}{4V_{dc}S/\pi} \quad (0 \leq M \leq 1). \quad (4)$$

It is necessary to determine four switching angles, namely  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  such that the equation sets (3) are satisfied. These equations are nonlinear. The whole interval of  $M$  may be categorized into three regions. In the first region, one solution set exists. In the second region, more than one solution set exists. In the last region, which this paper focuses on, there is no solution, naturally. Using intelligent algorithms such as: a GA leads to the minimization of harmonics instead of the elimination of them.

### B. Proposed Method Based on the Reduction of Eliminated Harmonics

As previously mentioned, the elimination of low order harmonics involves solving nonlinear transcendental equations. These equations are not solvable for some intervals of  $M$  (the third region). However, some loads such as electrical drives need to work in a wide range of  $M$  even in the third region. In these points, the desired harmonics cannot be completely eliminated. Instead, the minimization of harmonics leads to the presence of harmful harmonics such as the 5<sup>th</sup> order harmonic.

This paper purposes to neglect from elimination the harmonics with the highest order in Eq. (3). For example, elimination of the 11<sup>th</sup> harmonic is ignored. Therefore, only the 5<sup>th</sup> and the 7<sup>th</sup> harmonics are eliminated. As a result, one degree of freedom is added to Eq. (3). Four variables are dedicated to solving the three equations. This results in a set of equations which have feasible solutions for more intervals of  $M$ . For problem generalization, if neglecting the elimination of one harmonic cannot satisfy remaining equations, it is proposed to neglect the elimination of two harmonics. Subsequently, more harmonics are ignored until the remaining equations become solvable for the whole range of the modulation index. Since, the SHEPWM equations are nonlinear, the GA is proposed to solve them.

## IV. GENETIC ALGORITHM

As mentioned, SHEPWM equations are nonlinear. In order to solve these equations the genetic algorithm (GA), which is based on natural evolution and populations, is implemented.

This algorithm is usually used to reach a near global optimum solution. In each iteration of the GA a new set of strings, which are called chromosomes, with improved fitness is produced using genetic operators.

A more complete discussion of GAs including extensions to the general algorithm and related topics can be found in

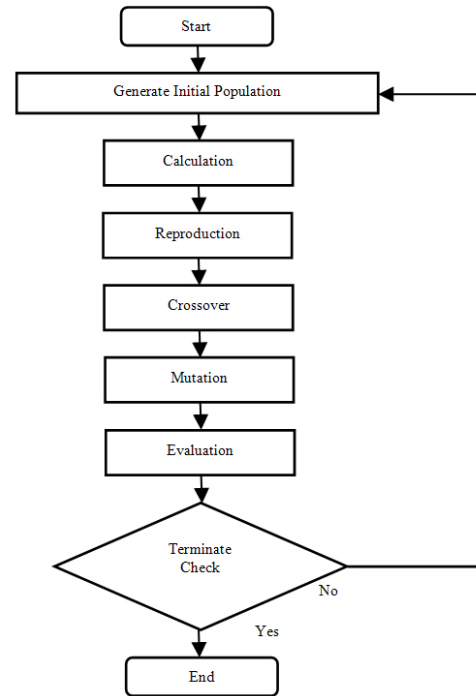


Fig. 3. Flowchart of Genetic Algorithm.

books by Davis [20], Goldberg [21], Holland [22], and Deb [23].

The structure of a simple GA consists mainly of three operators. A selection operator, a crossover operator which acts on a population of strings to perform the required reproduction and recombination, and a mutation operator which randomly alters character values, usually with a very low probability. The effect of these random alterations is to maintain diversity within the population, thereby preventing an early convergence of the algorithm to a possibly false peak.

### A. Chromosome Representation

In this study, each chromosome is taken as a possible solution for the problem, then each chromosome is developed based on single dimensional arrays with a length of  $S$ , where  $S$  is the number of angles.

### B. Initialization of the Population

For any GA it is necessary to initialize the population. The most common method is to randomly generate solutions for the entire population. All of the experiments discussed in this paper employ a completely random seeding of the initial population. Population size depends only on the nature of the problem and it must achieve a balance between the time complexity (consume for computing the fitness function and the genetic operators) and the search space measure. In this paper, the population size is set at 150.

### C. Reproduction

The degree of conformity of each object is calculated and an individual is reproduced under a fixed rule depending on the degree of conformity. Here, some individuals with a low degree of conformity will be screened, while individuals with a high degree of conformity will increase.

#### D. Crossover

New individuals are generated by the method of intersection that has been set up.

#### E. Mutation

This is performed by an operation determined by the installed mutation probability or mutation, and then a new individual is generated.

#### F. Evaluation of Fitness function

The fitness value is a measure of the appropriateness of a solution with respect to the original objective and the “amount of infeasibility”. The fitness function is formed by adding a penalty to the original objective function or fundamental component function [24], [25]. The GA process as well as an explanation is extracted from Refs. [26], [27].

These processes are presented in the flowchart in Fig. 3.

Since finding a feasible solution is a major problem, a penalty approach is used to decrease the fitness of infeasible solutions and toward the feasible region.

For each solution (or each chromosome), the fitness function is calculated as follows:

$$f = \min_{\theta_i} \left\{ \left( 100 \frac{V_1^* - V_1}{V_1^*} \right)^4 + \sum_{s=2}^S \frac{1}{h_s} \left( 50 \frac{V_{h_s}}{V_1} \right)^2 \right\}; \quad (5)$$

$$i = 1, 2, \dots, S$$

subject to:

$$0 \leq \theta_i \leq \pi/2 \quad (6)$$

where,  $V_1^*$  is the desired fundamental harmonic,  $S$  is the number of switching angles, and  $h_s$  is the order of the  $s^{\text{th}}$  viable harmonic at the output of a three phase multi-level inverter, e.g.  $h_2 = 5$  and  $h_4 = 11$ .

The main objective is to find a set of switching angles such that the magnitude of the fundamental harmonic reaches a desired value, i.e.  $V_1^*$ .  $M$  is a symbol representing  $V_1^*$  in Eq. (3). Whenever the fundamental harmonic violates its set point by more than 1%, the first term of Eq. (5) fines it by a power of 4 which is a very heavy penalty. As a result of using a power of 4, corresponding penalties for any deviation less than 1% gets a negligible value. Each harmonic ratio is weighted by the reciprocal of its harmonic order, i.e.  $1/h_s$ . As a result of this weighting method, reducing the low order harmonics gets a higher importance. It should be noted that, low order harmonics are more harmful and their filtering is more troublesome.

In this paper, the GA for each state is run 10 times, because it may fall into a local minima. The least fitness function between all runs is chosen. By increasing the number of runs, the probability of reaching the global minimum increases [9].

## V. SIMULATION RESULT

For implementing the proposed technique, a code based on the genetic algorithm is written in MATLAB software. All of the results are extracted from this code.

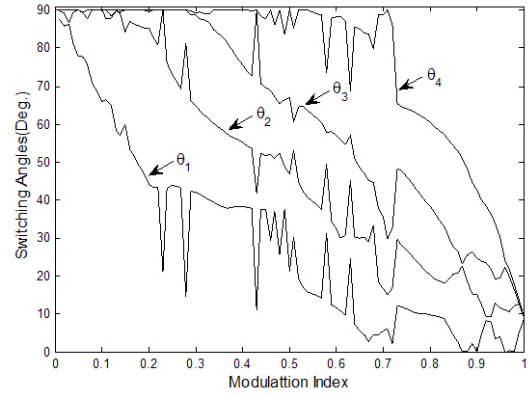


Fig. 4. Optimal switching angles versus M.

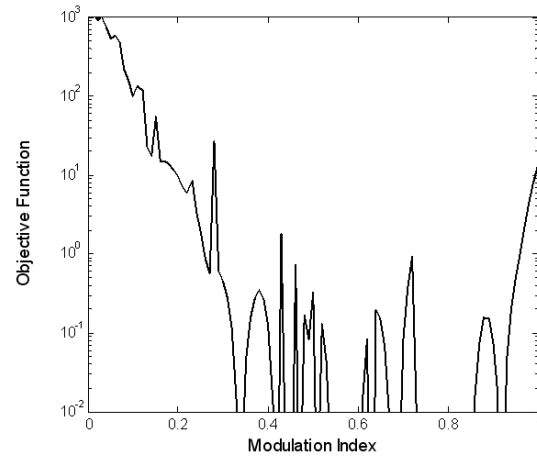


Fig. 5. Optimum values of objective function versus M.

First, the GA is applied to a 9-level inverter to eliminate the 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> harmonics as a base case. The curve of the switching angles versus M is shown in Fig. (4). M varies from zero to one [0-1]. The step of increasing M is 0.01. In some Ms (e.g. [0.4-0.7]), more than one solution exists. The GA falls randomly into one of them, so jerks are observed in this figure.

The value of the fitness function (Eq. 5) versus M is shown in Fig. (5). For values less or equal to  $10^{-2}$  it is supposed that the harmonics are eliminated. In other words, the values of the fitness function are the symbols of the Eq. (3). If a fitness value is less than  $10^{-2}$ , it means that this equation is solvable, otherwise it is unsolvable. In this figure, the whole range of M is not covered for harmonic elimination. The feasible Ms can be viewed as the set  $M = 0.34, [0.44 - 0.47], 0.51, [0.54 - 0.6], 0.63, [0.67 - 0.69], [0.73 - 0.85]$  and 0.92. In these regions, the quantity of the harmonics becomes zero.

Fig. (6) shows the values of low order harmonics and THD.

The desired fundamental component is satisfied for all Ms. In the feasible intervals of M, low order harmonics are omitted and THD has the least values.

In insoluble intervals, none of low order harmonics are eliminated.

To provide a proper voltage with an acceptable quality of harmonics especially in unsolvable Ms, this paper purposes neglecting the elimination of the highest order harmonic.

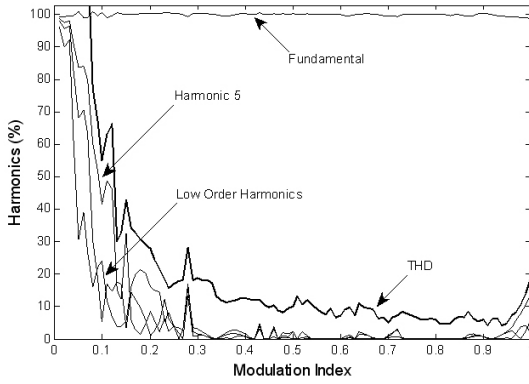


Fig. 6. Percentage of low order harmonics for base case.

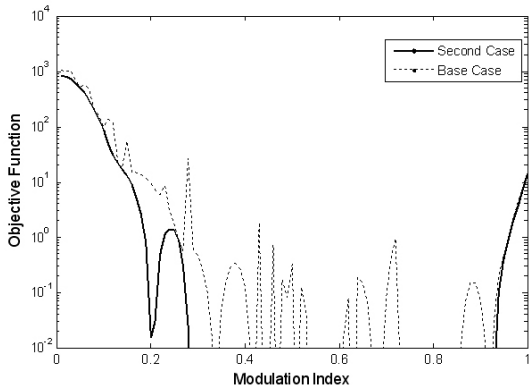


Fig. 7. Objective function for neglecting 11<sup>th</sup> Harmonic case.

Therefore, the 5<sup>th</sup> and 7<sup>th</sup> harmonics instead of the 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> harmonics are eliminated.

Fig. (7) shows the quantity of the fitness function for this case and compares it with the base case. The objective function values are less than the base case. This figure shows that the previous *M*s are solvable and that new solvable *M*s are added to the solvable set. Therefore, a continuous interval for solvable *M*s is obtained (i.e. 0.28-0.94). Also, it can be seen that some unsolvable *M*s still exist. The 5<sup>th</sup> and 7<sup>th</sup> harmonic values are compared between the two cases in Figs. (8) and (9), respectively. The solvable intervals in Figs. (8) and (9) are compatible with those in Fig. (7).

Fig. (10) shows the values of the 11<sup>th</sup> harmonic in the two cases. As predicted, the new case has more values than the base case.

As mentioned before, there are intervals of *M* for which SHEPWM equations are still not solvable. Elimination of more than one harmonic should be neglected. In the 9-level inverter, the 5<sup>th</sup> harmonic instead of the 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> harmonics is eliminated. Subsequently, if this case cannot satisfy the equations, a fourth case is performed, i.e. none of harmonics are eliminated and only the fundamental component meets the desired value. The objective functions of these four cases are compared in Fig. (12).  $Fit_{BaseCase}$  is devoted to the base case;  $Fit_{SecondCase}$  is devoted to the second case and so on. Thus, the whole interval is covered.

Finally, *M*s are divided into four sets. In the first set, the base case and the corresponding switching angles are chosen. In this case, all three harmonics are completely omitted. In the second set, the interval of 0.28-0.94 in spite of those devoted

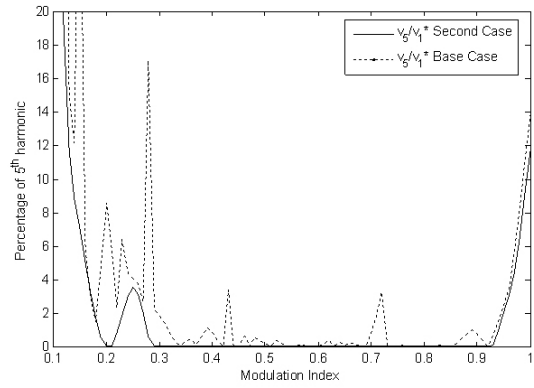


Fig. 8. Comparing 5<sup>th</sup> harmonic between two Cases.

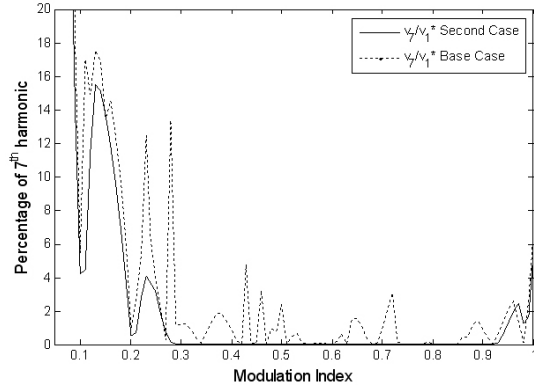


Fig. 9. Comparing 7<sup>th</sup> harmonic between two Cases.

to the base case are dedicated to the second case where the 5<sup>th</sup> and 7<sup>th</sup> harmonics are omitted. Subsequently, the intervals (0.15-0.28) and (0.94-0.96) are given for the third case. The remainder of the *M*s i.e. (0-0.15) and (0.96-1) are devoted to the last case. These regions are shown in Fig. (11).

For each *M*, the base case is implemented first. If it can satisfy all of the SHEPWM equations, it is acceptable. If not, the second case is implemented and so on. Finally, each *M* is dedicated to a corresponding region. Obviously, the *M* which is dedicated to the second region can belong to the third region too, but it is definitely related to the second case, because more harmonics are eliminated.

Table I shows some case studies in the interval of  $0 \leq M \leq 1$ . Two values based on the conventional and proposed methods are presented for the switching angles. Depending on the value of *M*, the related case of the proposed method is employed.

When the angles are equal, it means that *M* is in the base case.

Also, for *M*=0.72 the conventional SHEPWM is applied. Fig. 13 (a) shows the line voltage (the line voltage has more than 9 levels). Fig. 13 (b) shows the related frequency spectrum.

This *M* is located in the second case, so it is applied again to show the results. Fig. 14 (a) and (b) show the line voltage and the frequency spectrum of calculated angles in the second case. It can be seen that in Fig. 13 (b), all the low order harmonics exist, whereas in Fig. 14 (b), the 5<sup>th</sup> and 7<sup>th</sup> harmonics are removed.

This procedure is also implemented with an 11-level inverter. Notice that, there are five cases here. Fig. (15) compares

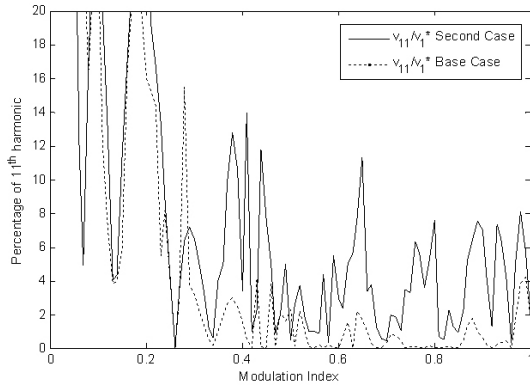


Fig. 10. Comparing 11<sup>th</sup> harmonic between two Cases.

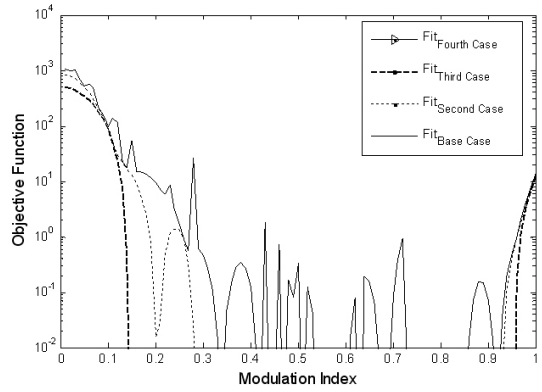


Fig. 12. Comparing objectives between four cases.

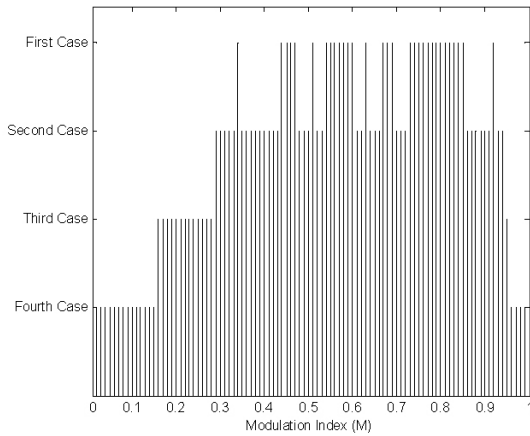
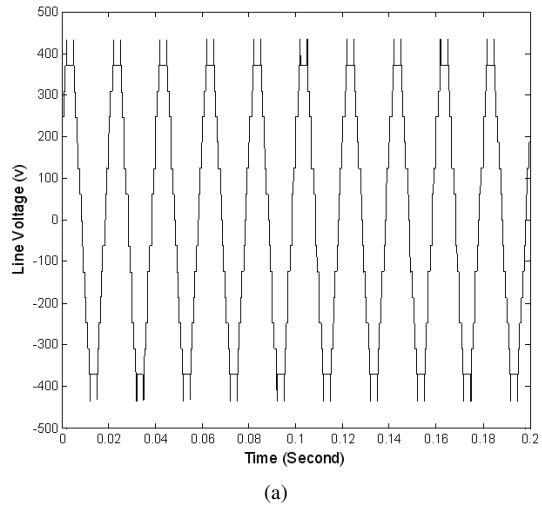


Fig. 11. Four regions versus Modulation Index.



the objective values of these cases. The same result is obtained.

Since a conventional GA is very time consuming, it is not an effective tool for on-line applications such as: dynamic operating. When running a GA it takes at least 3 second to reach a solution. Normally, a look-up table is used to save the pre-calculated angles. Then it orders the proper angles in a on-line application.

TABLE I  
CAES STUDIES

| $M^{des}$ |       | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ |
|-----------|-------|------------|------------|------------|------------|
| 0.1       | Con.  | 65.88      | 90.16      | 90.11      | 90.05      |
|           | Prop. | 67.53      | 89.47      | 89.54      | 89.96      |
| 0.2       | Con.  | 44.09      | 85.46      | 89.68      | 89.95      |
|           | Prop. | 46.93      | 86.47      | 87.20      | 89.60      |
| 0.3       | Con.  | 41.96      | 64.82      | 88.86      | 89.19      |
|           | Prop. | 40.53      | 65.12      | 89.04      | 89.83      |
| 0.4       | Con.  | 38.38      | 55.16      | 75.54      | 90.05      |
|           | Prop. | 38.97      | 54.37      | 76.04      | 90.04      |
| 0.5       | Con.  | 21.45      | 46.85      | 66.96      | 90.05      |
|           | Prop. | 24.98      | 51.21      | 63.31      | 89.00      |
| 0.6       | Con.  | 11.68      | 32.30      | 57.09      | 88.17      |
|           | Prop. | 11.68      | 32.30      | 57.09      | 88.17      |
| 0.7       | Con.  | 5.51       | 16.59      | 35.45      | 88.70      |
|           | Prop. | 11.34      | 35.57      | 47.42      | 70.72      |
| 0.8       | Con.  | 9.78       | 20.45      | 38.46      | 60.41      |
|           | Prop. | 9.78       | 20.45      | 38.46      | 60.41      |
| 0.9       | Con.  | 0.05       | 15.02      | 24.83      | 42.71      |
|           | Prop. | 10.82      | 13.97      | 20.42      | 44.77      |
| 1         | Con.  | 8.90       | 9.04       | 9.35       | 9.35       |
|           | Prop. | 3E-08      | 0.36       | 2.11       | 2.30       |

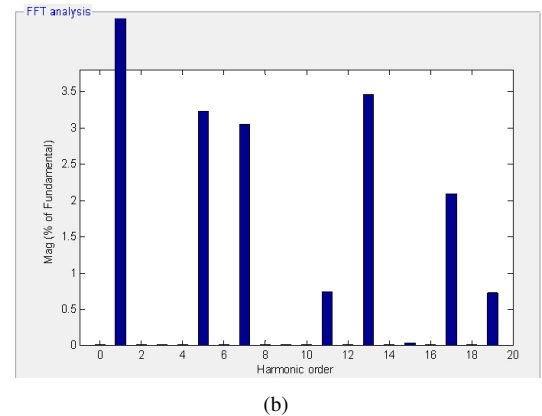


Fig. 13. Line voltage. (a) and Frequency Spectrum. (b) of a 9-level inverter for  $M=0.72$  based on base case.

## VI. CONCLUSION

In this paper, the SHEPWM switching strategy is developed to improve the quality of the low order harmonics in a wide range of modulation indexes. Normally, the equations related to SHEPWM do not have feasible solutions for all of the values of  $M$ . This leads to the presence of harmful harmonics such as: the 5<sup>th</sup> harmonic. It is suggested to reduce the number of eliminated harmonics. As a result, the 5<sup>th</sup> harmonic becomes zero for more values of  $M$ . Also, the equations are satisfied for all quantities of  $M$ .



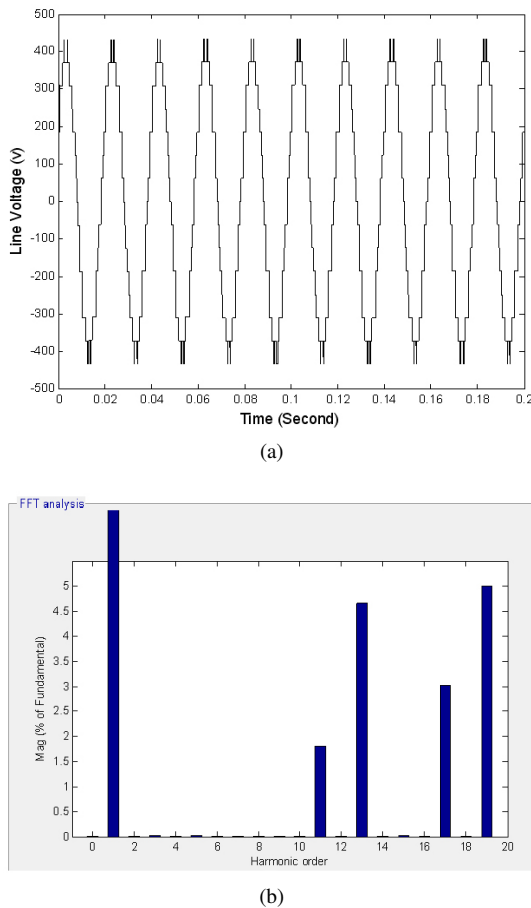


Fig. 14. Line voltage. (a) and Frequency Spectrum. (b) of a 9-level inverter for  $M=0.72$  based on Second case.

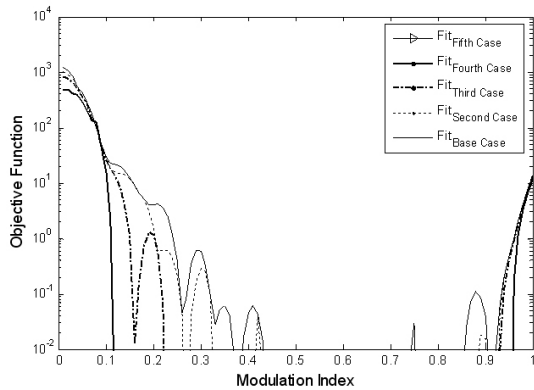


Fig. 15. Comparing objectives between five cases.

## REFERENCES

- [1] J. Rodriguez, J.-S. Lai, and F. Z. Peng, "Multilevel inverters: a survey of topologies, controls, and applications," *IEEE Trans. Ind. Electron.*, Vol. 49, No. 4, pp. 724-738, Aug. 2002.
- [2] C. Cecati, F. Ciancetta, P. Siano, "A multilevel inverter for PV systems with fuzzy logic control," *IEEE Trans. Ind. Electron.*, Vol. 57, No. 12, pp. 4115-4125, Dec. 2010.
- [3] N. Flourentzou, V. G. Agelidis, and G. D. Demetriades, "VSC-based HVDC power transmission systems: an overview," *IEEE Trans. Power Electron.*, Vol. 24, No. 3, pp. 592-602, Mar. 2009.
- [4] K. Gopakumar, A. Das, R. Ramchand, C. Patel, and K. Gopakumar, "A hybrid multilevel inverter topology for an open-end winding induction motor drive using two-level inverters in series with a capacitor fed H-bridge cell," *IEEE Trans. Ind. Electron.*, Vol. 57, No. 11, pp. 3707-3714, Nov. 2010.

- [5] N. Farokhnia, S. H. Fathi, and H. R. Toodeji, "Direct nonlinear control for individual DC voltage balancing in cascaded multilevel DSTATCOM," in *Proc. EPECS*, pp. 1-8, Nov. 2009.
- [6] D. G. Holmes and T. A. Lipo, *Pulse width modulation for power converters principles and practice*, NJ: Wiley-IEEE Press, Oct. 2003.
- [7] N. Yousefpoor, S. H. Fathi, N. Farokhnia, and S. H. Sadegh, "Application of OHSW technique in cascaded multi-level inverter with adjustable DC sources," *Proc. EPECS*, pp. 1-6, Nov. 2009.
- [8] W. Fei, X. Du, and B. Wu, "A generalized half-wave symmetry SHE-PWM formulation for multilevel voltage inverters," *IEEE Trans. Ind. Electron.*, Vol. 57, No. 9, pp. 3030-3038, Sep. 2009.
- [9] A. K. Kaviani, S. H. Fathi, N. Farokhnia, and A. J. Ardakani, "PSO, an effective tool for harmonics elimination and optimization in multi-level inverters," in *Proc. ICIEA*, pp. 2902-2907, May 2009.
- [10] S. Sirisukprasert, J.-S. Lai, and T.-H. Liu, "Optimum harmonic reduction with a wide range of modulation indexes for multilevel converters," *IEEE Trans. Ind. Electron.*, Vol. 49, No. 4, pp. 875-881, Aug. 2002.
- [11] J. Chiasson, L. Tolbert, K. McKenzie, and Z. Du, "Eliminating harmonics in a multi-level converter using the resultant theory of symmetric polynomials and resultants," *IEEE Trans. Control Syst. Technol.*, Vol. 13, No. 2, pp. 216-223, Mar. 2005.
- [12] K. El-Naggar and T. H. Abdelhamid, "Selective harmonic elimination of new family of multilevel inverters using genetic algorithms," *Energy Conversion and Management*, Vol. 49, No. 1, pp.89-95, Jan. 2008.
- [13] J. Rodriguez, S. Bernet, B. Wu, J. O. Pontt, and S. Kouro, "Multi-level voltage-source-converter topologies for industrial medium-voltage drives," *IEEE Trans. Ind. Electron.*, Vol. 54, No. 6, pp. 2930-2945, Dec. 2007.
- [14] S. Kouro, M. Malinowski, K. Gopakumar, J. Pou, L. G. Franquelo, B. Wu, J. Rodriguez, M. A. Pérez, and J. I. Leon, "Recent advances and industrial applications of multilevel converters," *IEEE Trans. on Ind. Electron.*, Vol. 57, No. 8, pp. 2553-2580, Aug. 2010.
- [15] M. Malinowski, K. Gopakumar, J. Rodriguez, and M. A. Pérez, "A survey on cascaded multilevel inverters," *IEEE Trans. Ind. Electron.*, Vol. 57, No. 7, pp. 2197-2206, Jul. 2009.
- [16] N. Farokhnia, S. H. Fathi, H. Vadizadeh, and H. Toodeji, "Comparison between approximate and accurate calculation of line voltage THD in multilevel inverters with unequal DC sources," in *Proc. ICIEA*, pp. 1034-1039, Jun. 2010.
- [17] N. Yousefpoor, N. Farokhnia, S. H. Fathi, J. S. Moghani, "Developed single-phase OMTHD technique for cascaded multi-level inverter By considering adjustable DC sources," in *Proc. EPECS*, pp. 1-6, Nov. 2009.
- [18] N. Yousefpoor, S. H. Fathi, N. Farokhnia, and H. A. Abyaneh, "Application of OMTHD on the line voltage of cascaded multi-level inverters with adjustable DC sources," in *Proc. ICIEA*, pp. 498-503, Jun. 2010.
- [19] M. G. Hosseini Aghdam, S. H. Fathi, and G. B. Gharehpetian, "Comparison of OMTHD and OHSW harmonic optimization techniques in multi-level voltage-source inverter with non-equal DC Sources," in *Proc. ICPE*, pp. 587-591, Oct. 2007.
- [20] L. Davis, *The Handbook of Genetic Algorithms*, New York, 1991.
- [21] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison Wesley, 1989
- [22] J. H. Holland, *Adaptation in natural and artificial systems*, The University of Michigan Press Ann Arbor, 1975
- [23] K. Deb, *Multi-Objective Optimization using Evolutionary Algorithms*, Wiley, 2001.
- [24] A. Homaiifar, C. X. Gi, and S. H. Lai, "Constrained optimization via genetic algorithms," *Simulation*, Vol. 62, No. 4, pp. 242-254, 1994.
- [25] Z. Michalewicz and M. Schoenauer, "Evolutionary algorithms for constrained parameter optimization problems" *Evolutionary Computation Journal*, Vol. 4, No. 1, pp. 1-32, 1996.
- [26] H. Min, C. S. Ko, and H. J. Ko, "The spatial and temporal consolidation of returned products in a closed-loop supply chain network," *Computers & Industrial Engineering*, Vol. 51, No. 2, pp. 309-320, Oct. 2006.
- [27] J. Rezaei and M. Davoodi, "A deterministic, multi-item inventory model with supplier selection and imperfect quality," *Applied Mathematical Modeling*, Vol. 32, pp. 2106-2116, 2008.



**Reza Salehi** was born in Sari, Iran, in 1985. He received his B.Sc from the Iran University of Science and Technology, Tehran, Iran, in 2008. He is currently working towards his M.Sc at the Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran. His research interests include multilevel inverters, power electronics and electrical drives.



**Mehrdad Abedi** received his B.Sc. in Electrical Engineering from the University of Tehran, Iran, in 1970, his MSc. from the Electrical Engineering Department, Imperial College, University of London, England, in 1973, and his Ph.D. in Electrical Engineering from Newcastle University, England, in 1977. He is currently a Professor in the Electrical Engineering Department of the Amirkabir University of Technology (AUT).



**Naeem Farokhnia** was born in Amol, Iran, in 1984. He received his B.Sc. and M.Sc. in Electrical Engineering from the Amirkabir University of Technology (Tehran Polytechnic), in 2006 and 2009, respectively, where he is currently working toward his Ph.D. in the Power Electronics Lab. His research interests include multilevel inverters, power electronics, FACTS devices and distributed generation.



**Seyed Hamid Fathi** received his B.Sc. and M.Sc. in Electrical Engineering from the Amirkabir University of Technology (AUT), Tehran, Iran and the Iran University of Science and Technology (IUST), Tehran, Iran, in 1984 and 1987, respectively. He received his Ph.D. in Electrical Engineering from the University of Newcastle Upon-Tyne, UK, in 1991. He then joined the Amirkabir University of Technology where he is presently an Associate Professor in the Department of Electrical Engineering. His research interests include power quality, flexible ac transmission systems (FACTS), power electronics and electric drives.