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Sliding Mode Controller for Torque and Pitch Control of PMSG Wind Power Systems

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Abstract

We propose a torque and pitch control scheme for variable speed wind turbines with permanent magnet synchronous generator (PMSG). A torque controller is designed to maximize the power below the rated wind speed and a pitch controller is designed to regulate the output power above the rated wind speed. The controllers exploit the sliding mode control scheme considering the variation of wind speed. Since the aerodynamic torque and rotor acceleration are difficult to measure in practice, a finite time convergent observer is designed which estimates them. In order to verify the proposed control strategy, we present stability analysis as well as simulation results.

Key Words: wind power system, PMSG, torque control, pitch control, sliding mode control

I. Introduction

Recently, environmental concerns are on the rise. In addition, because of the lack of fossil fuels, the interest in renewable energy sources rapidly grows. Especially wind is considered as one of the most efficient sources for generating electricity [1]. Wind power system transforms wind energy into electric energy through wind turbines and they are typically operated in two methods. One with fixed speed and the other with variable speed [2]. Despite its simple structure, fixed speed wind turbines have some problems with low efficiency, poor power quality, and limitation of utility. Therefore, they have been replaced by variable speed wind turbines in order to reduce mechanical fatigue and enhance efficiency. A permanent magnet synchronous generator (PMSG) is used for variable speed wind turbines because of its advantages such as simple structure and high efficiency.

For variable speed wind turbines which can limit the inlet wind power by adjusting the blade pitch angle there are mainly two controllers. One is the power controller regulating the output power and the other controlling the pitch angle of turbine blades (pitch controller). For wind turbine systems using PMSG, there exists a variety of control schemes such as PI control [3], [4], sliding mode control schemes, etc. Although the PI control scheme is widely used, it shows a limited performance, especially against uncertainties. Thus, various controllers including sliding mode control scheme are developed to provide better robustness. For power control, [5], [6] propose the sliding mode control scheme below the rated wind speed in order to maximize the generated power and [7], [8] introduce high order sliding mode controllers to reduce the chattering effect. For pitch control, most of the results use PI control scheme and recently a sliding mode pitch control scheme is presented in [9]. However, in [9] it is assumed that the aerodynamic torque and the wind speed are measurable, which is not realistic in practice.

This paper aims to provide controllers for the generator torque and the pitch angle of the wind power system based on PMSG. Both controllers are designed based on sliding mode control scheme without the assumption that the wind speed is measurable. Since the aerodynamic torque and the rotor acceleration are difficult to measure, an observer is introduced to estimate them. The observer is designed by using high order sliding mode scheme based on the supertwisting algorithm and provides estimate values of the aerodynamic torque (for torque controller) and the rotor acceleration (for pitch controller). To develop the sliding mode pitch control, the wind power system is linearized with respect to the pitch angle relying on the direct feedback linearizing (DFL) theory [10]. This approach provides robustness with respect to external disturbances and uncertain dynamics.

This paper is organized as follows. In Section II, the wind turbine generator model is presented. Section III introduces an observer and the proposed controller which achieve the objectives of the torque control and pitch control. In Section IV,

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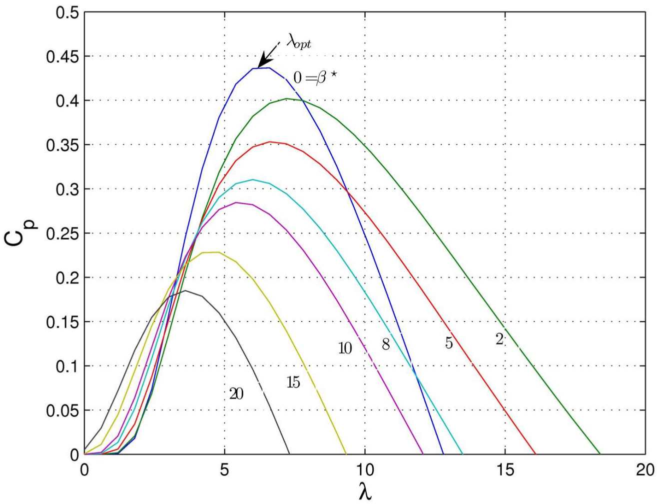

Fig. 1. Wind turbine power coefficient.
($\beta^\star$ is the pitch angle maximizing the power coefficient)

the proposed controllers are verified by simulations. Finally, Section V concludes the paper.

II. Wind Turbine Generator Model

A. Wind Turbine Rotor Model

The aerodynamic power P_w produced by wind turbine rotor is given by

$$P_w = \frac{\rho \pi r^2 v^3 C_p(\lambda, \beta)}{2} \tag{1}$$

where ρ is the air density, r is the radius of rotor, v is the wind speed, C_p is the power coefficient, λ is the tip speed ratio, and β is the pitch angle. The tip speed ratio λ is defined as

$$\lambda = \frac{r\omega_m}{v}$$

where ω_m is the angular rotor speed.

The power coefficient model $C_p(\lambda, \beta)$ used in our paper is taken from [11] and given by

$$\begin{aligned} C_p(\lambda, \beta) &= c_1 \left(c_2 \frac{1}{\Gamma} - c_3\beta - c_4\beta^x - c_5 \right) e^{-\frac{c_6}{\Gamma}} \\ \frac{1}{\Gamma} &= \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{1+\beta^3} \end{aligned} \tag{2}$$

where the coefficients $c_1, \ldots, c_6$ and x depend on wind turbines. In Fig. 1, C_p is plotted with respect to the tip speed ratio λ for various values of the pitch angle β. The values for $c_1, \ldots, c_6$ and x used in the paper are given in Appendix. Using (1), the aerodynamic torque T_w is given by

$$T_w = \frac{P_w}{\omega_m} = \frac{\rho \pi r^3 v^2 C_p(\lambda, \beta)}{2\lambda}.$$

B. Drive Train and PMSG Model

We consider a one-mass drive train model given by

$$\dot{\omega}_e = \frac{P}{J}\left(\frac{T_w}{N} + T_e \right) \tag{3}$$

where $\omega_e = P\omega_g = PN\omega_m$ is the electrical rotor speed, ω_g is the mechanical rotor speed, P is the number of pole pairs, J is the inertia of turbine, N is the gear ratio, and T_e is the generator electromagnetic torque. Note that in our setting, T_e is negative for generating.

The voltage and torque equations of PMSG in the synchronous reference frame [4] are given by

$$\begin{aligned} u_d &= R_s i_d + \frac{d}{dt} L_d i_d - \omega_e L_q i_q \\ u_q &= R_s i_q + \frac{d}{dt} L_q i_q + \omega_e (L_d i_d + \phi) \\ T_e &= \frac{3}{2} P((L_d - L_q) i_d i_q + \phi i_q) \end{aligned}$$

where u_d, u_q are the dq axis voltages, i_d, i_q are dq axis currents, R_s is the stator resistance, L_d, L_q are the dq axis inductances, and ϕ is the permanent magnetic flux. If the PMSG has a uniform air gap, then $L_d = L_q = L$ and voltage equations can be expressed with respect to currents.

$$\dot{i}_d = -\frac{R_s}{L} i_d + \omega_e i_q + \frac{1}{L} u_d \tag{4}$$

$$\dot{i}_q = -\frac{R_s}{L} i_q - \omega_e i_d - \frac{\omega_e \phi}{L} + \frac{1}{L} u_q \tag{5}$$

$$T_e = \frac{3}{2} P \phi i_q.$$

III. Controller Design

In this section, we propose a torque controller and a pitch controller using the sliding mode control scheme. At first, an observer is developed to estimate the aerodynamic torque and rotor acceleration. Secondly, a torque controller is designed for two objectives. One objective is to reduce the copper loss by setting the d axis current to be zero and the other is to maximize the generating power by tracking the maximum power operating point. Two sliding surfaces are designed for these objectives. In the case of the pitch controller, a sliding surface is designed to maintain the rated rotor speed so that the generated power is regulated.

From the equations (3), (4) and (5), the wind power system is represented by

$$\begin{aligned} \dot{i}_d &= -\frac{R_s}{L} i_d + \omega_e i_q + \frac{1}{L} u_d \\ \dot{i}_q &= -\frac{R_s}{L} i_q - \omega_e i_d - \frac{\omega_e \phi}{L} + \frac{1}{L} u_q \\ \dot{\omega}_e &= \frac{P}{J}\left(\frac{T_w}{N} + T_e \right) \end{aligned} \tag{6}$$

where the dq axis voltages are control inputs and i_d, i_q, and ω_e are state variables.

We assume that the system (6) satisfies the following conditions.

Assumption 1: The wind speed and pitch angle is in the range of $0 < \underline{v} \le v \le \overline{v}$, $\beta^\star \le \beta \le 90°$, respectively. Moreover, there exist positive constants W_0, W_1, B, and V_1 such that $|\omega_e| < W_0$, $|\frac{d\omega_e}{dt}| < W_1$, $|\frac{d\beta}{dt}| < B$, and $|\frac{dv}{dt}| < V_1$. $\diamond$

Assumption 1 is technical and the bounds are used to derive the control laws. The wind power system is operated in the specific range of wind speed. In practice, if the wind speed is out of some range $\underline{v} \leq v \leq \overline{v}$, then the system will be shut down, or does not generate any power. In addition, in order to generate maximum power and to keep the rated power, the pitch angle (in degree) should be between $\beta^\star$ and $90°$. The pitch angle $\beta = 90°$ means that the wind power system is shut down so that no power is generated. It is assumed that the time derivatives of the rotor speed, pitch angle, and wind speed are bounded. It is also assumed that the rotor speed is bounded.

A. Observer Design

The estimated aerodynamic torque and rotor acceleration are used in the torque controller and the pitch controller. In order to estimate them, we use the sliding mode observer [8] based on the supertwisting algorithm [12].

$$\begin{aligned}
\dot{\hat{w}}_e &= \frac{P}{J}T_e + u_1 - \mu|e|^{\frac{1}{2}}\operatorname{sgn}(e),\ e := \hat{\omega}_e - \omega_e \\
\dot{u}_1 &= \begin{cases} -u, & |u| > U_M \\ -\alpha\operatorname{sgn}(e), & |u| \leq U_M \end{cases} \\
u &= u_1 - \mu|e|^{\frac{1}{2}}\operatorname{sgn}(e), \quad \hat{T}_w = \frac{JN}{P}u_1.
\end{aligned} \tag{7}$$

From (3) and (7), the dynamics of e is derived as

$$\dot{e} = -\frac{P}{JN}T_w + u_1 - \mu|e|^{\frac{1}{2}}\operatorname{sgn}(e).$$

According to supertwisting algorithm, e and $\dot{e}$ converge to zero in finite time, and thus T_w, ω_e can be estimated. U_M, α, and μ are the positive constants satisfying the following equations.

$$\begin{aligned}
qU_M &> \left|\frac{P}{JN}T_w\right|, \quad 0 < q < 1, \quad \alpha > C \\
C &\geq \left|\frac{P}{JN}\dot{T}_w\right|, \quad \mu > \sqrt{\frac{2}{(\alpha - C)}\frac{(\alpha + C)(1+q)}{(1-q)}}.
\end{aligned}$$

In order to apply the supertwisting algorithm, T_w and $\dot{T}_w$ should be bounded, which is assured by Lemma 1.

Lemma 1: Suppose that there exists a sufficiently small positive constant $w_e^\star$ such that $\omega_e \geq \omega_e^\star$. Then there exist constants T_0 and T_1 so that $|T_w| < T_0$ and $|\dot{T}_w| < T_1$. ◇

Proof: The constant T_0 can be found as follows. Represent T_w as

$$T_w = \frac{1}{2}\frac{\rho\pi r^2 v^3 NPC_p(\lambda, \beta)}{\omega_e}.$$

Since C_p is bounded by C_p^* (see item 1) in Appendix), we have

$$|T_w| \leq \frac{1}{2}\frac{\rho\pi r^2 \overline{v}^3 NPC_p^*}{\omega_e^\star} =: T_0.$$

To find T_1, we first note that T_w is a function of ω_e, β and v. Note also that λ is a function of ω_e and v. Then,

$$\begin{aligned}
\dot{T}_w &= \frac{\partial T_w}{\partial \omega_e}\dot{\omega}_e + \frac{\partial T_w}{\partial \beta}\dot{\beta} + \frac{\partial T_w}{\partial v}\dot{v} \\
&= \left(-AC_p\frac{1}{\omega_e^2} + \frac{A}{\omega_e}\frac{\partial C_p}{\partial \omega_e}\right)\dot{\omega}_e \\
&\quad + \frac{A}{\omega_e}\frac{\partial C_p}{\partial \beta}\dot{\beta} + \frac{3AC_p}{\omega_e v}\dot{v} + \frac{A}{\omega_e}\frac{\partial C_p}{\partial v}\dot{v}
\end{aligned}$$

where $A = \frac{\rho\pi r^2 v^3 NP}{2}$. According to Assumption 1, it holds that $|\frac{d\omega_e}{dt}| < W_1$, $|\frac{d\beta}{dt}| < B$, and $|\frac{dv}{dt}| < V_1$. Partial derivatives of C_p are computed in Appendix. We can also find constants K_1, K_2, and K_3 such that $|\frac{\partial C_p}{\partial \omega_e}| < K_1$, $|\frac{\partial C_p}{\partial \beta}| < K_2$, and $|\frac{\partial C_p}{\partial v}| < K_3$. Thus, $|\dot{T}_w|$ is bounded by T_1 which is given by

$$T_1 = \frac{A_{\max}}{\omega_e^\star}\left[\left(\frac{C_p^*}{\omega_e^\star} + K_1\right)W_1 + K_2 B + \left(\frac{3C_p^*}{\underline{v}} + K_3\right)V_1\right].$$

where $A_{\max} = \frac{\rho\pi r^2 \overline{v}^3 NP}{2}$. ■

B. Torque Controller Design

The torque controller should be designed to maximize the generating power below the rated wind speed. To achieve this, C_p should maintain the maximum value $C_p(\lambda_{opt}, \beta^\star)$ in Fig. 1. Thus the pitch angle β is set to be $\beta^\star$. At the maximum value of C_p, the aerodynamic torque is expressed as

$$\begin{aligned}
T_{wopt} &= \frac{\rho\pi r^5 C_p(\lambda_{opt}, \beta^\star)}{2N^2P^2\lambda_{opt}^3}\omega_{eopt}^2 = K_{opt}\omega_{eopt}^2 \\
K_{opt} &= \frac{\rho\pi r^5 C_p(\lambda_{opt}, \beta^\star)}{2N^2P^2\lambda_{opt}^3}, \quad \omega_{eopt} = \frac{NPv\lambda_{opt}}{r}
\end{aligned}$$

where K_{opt} is the constant value and ω_{eopt} is the rotor speed when the wind turbine generates the maximum power at a given wind speed. Note that ω_{eopt} is a function of the wind speed v. In order to track the maximum power point, the generator torque T_e should track the optimal aerodynamic torque T_{wopt} considering the gear ratio.

Thus the sliding surface for the torque controller is designed as follows.

$$\begin{aligned}
S_d &= \gamma_1(i_{dref} - i_d) \\
S_q &= K_{opt}\omega_e^2 + \frac{3}{2}NP\phi i_q
\end{aligned}$$

where $\gamma_1 < 0$, i_{dref} is set to zero for minimizing copper loss and S_q is to maximize the generating power. If the system trajectory reaches the sliding surfaces and the rotor speed converges to its optimal value, then the objective will be achieved.

The sliding mode torque controller is proposed as follows.

Theorem 1: Suppose that there exists a sufficiently small positive constant $\omega_e^\star$ such that $\omega_e(t) \geq \omega_e^\star, \forall t \geq 0$ for system (6). Then the sliding mode controller

$$u_d = R_s i_d - \omega_e L i_q + K_d\operatorname{sgn}(S_d) \tag{8}$$

$$\begin{aligned}
u_q &= -\frac{4K_{opt}\omega_e L}{3NP\phi}\frac{P}{J}\left(\frac{\hat{T}_w}{N} + \frac{3}{2}P\phi i_q\right) \\
&\quad + R_s i_q + \omega_e L i_d + \omega_e\phi - K_q\operatorname{sgn}(S_q)
\end{aligned} \tag{9}$$

with $K_d > 0$ and $K_q > 0$, renders the closed loop system trajectory reach the sliding surfaces S_d and S_q in finite time. ◇

Proof: We choose the Lyapunov function

$$V = \frac{1}{2}S_d^2 + \frac{1}{2}S_q^2.$$

Taking time derivative of V yields

$$\begin{aligned}\dot{V} =& S_d\dot{S}_d + S_q\dot{S}_q \\ =& -S_d\gamma_1\left(-\frac{R_s}{L}i_d + \omega_e i_q + \frac{u_d}{L}\right) \\ &+ S_q\left[2K_{opt}\omega_e\frac{P}{J}\left(\frac{T_w}{N} + \frac{3}{2}P\phi i_q\right)\right. \\ &\left.+\frac{3}{2}NP\phi\left(-\frac{R_s}{L}i_q - \omega_e i_d - \frac{\omega_e}{L}\phi + \frac{u_q}{L}\right)\right].\end{aligned} \tag{10}$$

Substituting the control (8) and (9) into (10), one has

$$\begin{aligned}\dot{V} =& -\gamma_1\frac{K_d}{L}|S_d| \\ &+ S_q\frac{2K_{opt}\omega_e P}{NJ}(T_w - \hat{T}_w) - \frac{3}{2}NP\phi\frac{1}{L}K_q|S_q|.\end{aligned} \tag{11}$$

Because the pitch angle β is fixed at $\beta^\star$, the conditions of Lemma 1 which are essential for operating observer properly are satisfied. Thus, $T_w - \hat{T}_w$ in equation (11) becomes zero in finite time. Hence,

$$\dot{V} = -\gamma_1\frac{K_d}{L}|S_d| - \frac{3}{2}NP\phi\frac{1}{L}K_q|S_q|.$$

From the fact that $K_d > 0$ and $K_q > 0$, it follows that $\dot{V} < 0$ when $S_d \neq 0$ and $S_q \neq 0$. Therefore, the system trajectory reaches the sliding surface S_d and S_q in finite time. ■

Lemma 2: Suppose that v is unknown but constant. Suppose also that there exists $\lambda^\star$ such that $\lambda^\star = \max\{\lambda \in \mathbb{R} | \lambda < \lambda_{opt}, \frac{C_p(\lambda,\beta^\star)}{\lambda^3} = \frac{C_p(\lambda_{opt},\beta^\star)}{\lambda_{opt}^3}\}$ and let $\underline{\omega}_e = \frac{NPv\lambda^\star}{r}$. Consider the system (6) with the control given in Theorem 1 and suppose that $\omega_e(t) > \underline{\omega}_e, \forall t \geq 0$. If the system trajectory reaches the sliding surfaces S_d and S_q and remains there, then $\omega_e(t)$ asymptotically converges to ω_{eopt}. ◇

Remark 1: The existence of the $\lambda^\star$ used in Lemma 2 depends on the parameters in (2). Noting that $\beta^\star = 0$ in our case, one can prove analytically (or numerically) that $\frac{C_p(\lambda,\beta^\star)}{\lambda^3} = \frac{C_p(\lambda_{opt},\beta^\star)}{\lambda_{opt}^3}$ has two distinct real roots for the parameters $c_1, \ldots, c_6$ given in Appendix, which means that the assumption holds true.

Proof: We choose the Lyapunov function $V = \frac{1}{2}(\omega_e - \omega_{eopt})^2$. Assuming that the trajectory is on the sliding surfaces and taking time derivative of V yield

$$\dot{V} = \frac{P}{JN}(\omega_e - \omega_{eopt})(K - K_{opt})\omega_e^2,$$

where we used the assumption that v is constant and K is given by $K = \frac{1}{2}\frac{\rho\pi r^5}{N^2P^2}\frac{C_p(\lambda,\beta^\star)}{\lambda^3}$. To determine $\dot{V} < 0$ for $\omega_e > \underline{\omega}_e$, we consider two cases.

Case 1. $\omega_e - \omega_{eopt} > 0$:
We first note that $\omega_{eopt} > 0$ by Assumption 1 and that $\lambda > \lambda_{opt}$. After simple computation, one sees that $C_p(\lambda, \beta^\star) < C_p(\lambda^*, \beta^\star)$ when $\lambda \neq \lambda_{opt}$ and hence

$$\frac{C_p(\lambda,\beta^\star)}{\lambda^3} < \frac{C_p(\lambda_{opt},\beta^\star)}{\lambda^3} < \frac{C_p(\lambda_{opt},\beta^\star)}{\lambda_{opt}^3}$$

which means $K - K_{opt} < 0$ and that $\dot{V} < 0$.

Case 2. $\omega_e - \omega_{eopt} < 0$:
By assumption we have $\underline{\omega}_e < \omega_e < \omega_{eopt}$ and $\lambda^\star < \lambda < \lambda_{opt}$. It is a simple matter of computation to have $\frac{d}{d\lambda}\frac{C_p(\lambda,\beta^\star)}{\lambda^3} = \frac{a_3}{\lambda^6}e^{-1/\lambda}(a_2\lambda^2 - a_1\lambda + a_0)$ with $a_i > 0$. Let λ_1, λ_2 with $\lambda_1 < \lambda_2$ be two roots of $a_2\lambda^2 - a_1\lambda + a_0 = 0$. Noting that $\frac{d}{d\lambda}\frac{C_p(\lambda,\beta^\star)}{\lambda^3} = \frac{1}{\lambda^4}\left(\lambda\frac{dC_p(\lambda,\beta^\star)}{d\lambda} - 3C_p(\lambda,\beta^\star)\right)$, one has $\frac{d}{d\lambda}\frac{C_p(\lambda,\beta^\star)}{\lambda^3}|_{\lambda_{opt}} < 0$ since $\frac{dC_p(\lambda,\beta^\star)}{d\lambda}|_{\lambda_{opt}} = 0$. This implies that $\lambda_1 < \lambda_{opt} < \lambda_2$ and that, by continuity, there exists some $\lambda^\dagger$ with $\lambda^\star < \lambda^\dagger < \lambda_{opt}$ such that $\frac{C_p(\lambda^\dagger,\beta^\star)}{\lambda^{\dagger 3}} > \frac{C_p(\lambda_{opt},\beta^\star)}{\lambda_{opt}^3}$. Thus, from the definition of $\lambda^\star$, it follows that $K - K_{opt} > 0$ for $\lambda^\star < \lambda < \lambda_{opt}$. Therefore, $\dot{V} < 0$ when $\underline{\omega}_e < \omega_e < \omega_{eopt}$.

From the results considered in the cases 1 and 2, one has $\dot{V} < 0$ whenever $\underline{\omega}_e < \omega_e$, and as a result, $\omega_e(t)$ asymptotically approaches ω_{eopt}. ■

From Theorem 1 and Lemma 2, it follows that the wind power system with the controller given by (8) and (9) can generate maximum power provided that $\omega_e(t) > \max\{\underline{\omega}_e, \omega_e^\star\}$ and v is constant.

C. Pitch Controller Design

Assuming that the torque control works well, the generating power is given by

$$P_e = T_e\omega_g = T_e\frac{\omega_e}{P} = -\frac{K_{opt}\omega_e^3}{NP}.$$

From this equation, we note that the generating power is determined by the rotor speed ω_e. When the wind speed is above the rated speed, the power should be regulated and this is done by the pitch controller. In order to regulate the power, we control the pitch angle β so that ω_e approaches the rated value $\omega_e^\dagger$, which means that P_e converges to its rated value. Hence the target value of P_e is given by

$$P_e^\dagger = -\frac{K_{opt}{\omega_e^\dagger}^3}{NP}.$$

We design a pitch controller so that it only operates in the range of $\omega_e > \omega_e^\dagger$.

Now, we consider the drive train model which contains β. If the torque control performs well, then the rotor dynamics becomes

$$\begin{aligned}\dot{\omega}_e &= \frac{P}{J}\left(\frac{T_w}{N} + T_e\right) \\ &= \frac{P}{J}\left(\frac{1}{2}\frac{\rho\pi r^2 v^3 NPC_p(\lambda,\beta)}{\omega_e N} - \frac{K_{opt}\omega_e^2}{N}\right).\end{aligned} \tag{12}$$

The pitch angle dynamics, which is modeled as a first order system, is given by

$$\frac{d\beta}{dt} = \frac{1}{T_\beta}(u_\beta - \beta)$$

where u_β is the control to be designed.

In order to use the sliding mode control, we linearize the equation (12) with respect to the pitch angle β. To do this, we define

$$x_1 = \omega_e$$
$$x_2 = \dot{\omega}_e = \frac{P}{J}\left(\frac{T_w}{N} + T_e\right) = \frac{P}{JN}\left(T_w - K_{opt}\omega_e^2\right).$$

Then one has

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \frac{P}{JN}\Big(- AC_p\frac{x_2}{x_1^2} + \frac{Ax_2}{x_1}\frac{\partial C_p}{\partial x_1} - \frac{A\beta}{x_1 T_\beta}\frac{\partial C_p}{\partial \beta}\\ &\quad + \frac{3AC_p}{vx_1}\dot{v} + \frac{A}{x_1}\frac{\partial C_p}{\partial v}\dot{v} - 2K_{opt}x_1x_2\Big) \\ &\quad + \frac{PA}{JNT_\beta x_1}\frac{\partial C_p}{\partial \beta}u_\beta\end{aligned} \tag{13}$$

where $A = \frac{\rho\pi r^2 v^3 NP}{2}$.

We make the following assumption, which is true for the parameters used in this paper.

Assumption 2: When $x_1 > \omega_e^\dagger$, $\frac{\partial C_p}{\partial \beta} < 0$ and there exists a constant $\bar{C}_{p\beta}$ such that $0 < \bar{C}_{p\beta} < |\frac{\partial C_p}{\partial \beta}|$. $\diamond$

Before proceeding, let $\hat{v} = (\underline{v} + \overline{v})/2$ which is assumed to be known and define

$$\hat{A} = \frac{\rho\pi r^2 \hat{v}^3 NP}{2}$$
$$\hat{C}_p = C_p(\lambda, \beta)|_{v=\hat{v}}$$
$$\hat{x}_2 = \frac{P}{JN}(\hat{T}_w - K_{opt}\hat{\omega}_e^2).$$

We define the sliding surface for the pitch controller as follows.

$$S_\beta = x_2 + \gamma_2(x_1 - \omega_e^\dagger)$$

where $\gamma_2 > 0$ is constant. The sliding surface S_β is designed so that the rotor speed converges to its rated value and remains still. The proposed sliding mode pitch controller is given by

$$u_\beta = \beta - \frac{\hat{a}_1 + \hat{a}_2}{\hat{b}} - K_\beta \text{sgn}(\hat{S}_\beta) \tag{14}$$

where

$$\hat{a}_1 = -\frac{P}{JN}2K_{opt}x_1\hat{x}_2 + \gamma_2\hat{x}_2$$
$$\hat{a}_2 = -\frac{P}{JN}\hat{A}\hat{C}_p\frac{1}{x_1^2}\hat{x}_2 + \frac{P}{JN}\frac{\hat{A}}{x_1}\hat{x}_2\frac{\partial \hat{C}_p}{\partial x_1}$$
$$\hat{b} = \frac{P\hat{A}}{JNT_\beta x_1}\frac{\partial \hat{C}_p}{\partial \beta}$$
$$\hat{S}_\beta = \hat{x}_2 + \gamma_2(x_1 - \omega_e^\dagger).$$

Note that K_β is the controller gain to be chosen later and that $\hat{T}_w$ is defined in (7).

Now we take the sliding mode controller gain K_β as follows. Firstly, define

$$h = a_1 + a_2 - \frac{\hat{a}_1 + \hat{a}_2}{\hat{b}}b$$

where

$$a_1 = -\frac{P}{JN}2K_{opt}x_1x_2 + \gamma_2 x_2$$
$$\begin{aligned}a_2 &= -\frac{P}{JN}AC_p\frac{1}{x_1^2}x_2 + \frac{P}{JN}\frac{A}{x_1}x_2\frac{\partial C_p}{\partial x_1}\\ &\quad + \frac{P}{JN}\frac{3AC_p}{vx_1}\dot{v} + \frac{P}{JN}\frac{A}{x_1}\frac{\partial C_p}{\partial v}\dot{v}\end{aligned}$$
$$b = \frac{PA}{JNT_\beta x_1}\frac{\partial C_p}{\partial \beta}.$$

We note that by Assumption 1, a_1, a_2 and b are bounded and we can compute

$$\begin{aligned}\frac{|h|}{b} = -\left|\frac{a_1 + a_2}{b} - \frac{\hat{a}_1 + \hat{a}_2}{\hat{b}}\right| &\geq -\left|\frac{a_1 + a_2}{b}\right| - \left|\frac{\hat{a}_1 + \hat{a}_2}{\hat{b}}\right|\\ &\geq \sigma\end{aligned}$$

where

$$\begin{aligned}\sigma = &-\frac{T_\beta W_0}{\bar{C}_{p\beta}}\left(\frac{2K_{opt}W_0W_1}{A_{\min}} + C_p^*\frac{W_1}{{\omega_e^\dagger}^2} + \frac{W_1}{\omega_e^\dagger}K_1\right.\\ &\left.+\frac{3C_p^*}{\underline{v}\omega_e^\dagger}V_1 + \frac{K_3V_1}{\omega_e^\dagger} + \frac{JN\gamma_2W_1}{PA_{\min}}\right)\\ &- \frac{T_\beta W_0 W_1}{\bar{C}_{p\beta}}\left(\frac{2K_{opt}W_0}{\hat{A}} + C_p^*\frac{1}{{\omega_e^\dagger}^2} + \frac{1}{\omega_e^\dagger}K_1 + \frac{JN\gamma_2}{P\hat{A}}\right)\end{aligned}$$

with $A_{\min} = \frac{\rho\pi r^2 \underline{v}^3 NP}{2}$ and C_p^* defined in Appendix. Note that K_1 is the bound for $|\frac{\partial C_p}{\partial \omega_e}|$ (see the proof of Lemma 1). Finally we choose K_β such that

$$K_\beta < \sigma.$$

The following theorem states the convergence result.

Theorem 2: Consider the system (13) and suppose that $\omega_e > \omega_e^\dagger$. Under Assumptions 1 and 2, the sliding mode controller u_β given by (14) renders the closed loop system trajectory reach the sliding surface S_β in finite time. Moreover, it holds that $\omega_e \to \omega_e^\dagger$ and $\dot{\omega}_e \to 0$ as $t \to \infty$. $\diamond$

Proof: We choose the Lyapunov function $V = \frac{1}{2}S_\beta^2$. Taking time derivative of V yields

$$\begin{aligned}\dot{V} =& S_\beta\dot{S}_\beta\\ =& S_\beta\Big[\frac{P}{JN}\Big(- AC_p\frac{1}{x_1^2}x_2 + \frac{A}{x_1}x_2\frac{dC_p}{dx_1}\\ &- \frac{A\beta}{x_1T_\beta}\frac{dC_p}{d\beta} + \frac{3AC_p}{vx_1}\frac{dv}{dt} + \frac{A}{x_1}\frac{dC_p}{dv}\frac{dv}{dt}\\ &-2K_{opt}x_1x_2\Big) + \gamma_2x_2 + \frac{PA}{JNT_\beta x_1}\frac{dC_p}{d\beta}u_\beta\Big]\\ =& S_\beta(a_1 + a_2 - b\beta + bu_\beta).\end{aligned}$$

Applying the control input (14), we have

$$\begin{aligned}\dot{V} =& S_\beta \left(a_1 + a_2 - \frac{\hat{a}_1 + \hat{a}_2}{\hat{b}} b - bK_\beta \text{sgn}(\hat{S}_\beta) \right) \\ =& S_\beta \left(h - bK_\beta \text{sgn}(\hat{S}_\beta) \right) \\ =& S_\beta h + \hat{S}_\beta h - \hat{S}_\beta h - S_\beta bK_\beta \text{sgn}(\hat{S}_\beta) \\ &+ \hat{S}_\beta bK_\beta \text{sgn}(\hat{S}_\beta) - \hat{S}_\beta bK_\beta \text{sgn}(\hat{S}_\beta) \\ =& (\hat{x}_2 - x_2) \left(bK_\beta \text{sgn}(\hat{S}_\beta) - h \right) \\ &+ \hat{S}_\beta h - \hat{S}_\beta bK_\beta \text{sgn}(\hat{S}_\beta) \\ \le& |bK_\beta \text{sgn}(\hat{S}_\beta) - h||\hat{x}_2 - x_2| + (\sigma - K_\beta) b|\hat{S}_\beta|.\end{aligned}$$

In the above equation, $\hat{x}_2 - x_2$ is the derivative of the observation error. Because the pitch controller is only operated in the region of $\omega_e > \omega_e^\dagger$ and β is limited by Assumption 1, the conditions of Lemma 1 are satisfied. Thus $\hat{x}_2 - x_2$ becomes zero in finite time. After that instant, it holds that $\hat{S}_\beta = S_\beta$ since $\hat{x}_2 = x_2$. Finally, it follows that

$$\dot{V} \le (\sigma - K_\beta) b |S_\beta|.$$

Because $\dot{V} < 0$ except at $S_\beta = 0$, the system trajectory reaches the sliding surface $S_\beta = 0$ in finite time. After that instant, the dynamics of the system is reduced to

$$\dot{x}_1 = -\gamma_2 (x_1 - \omega_e^\dagger).$$

As a result, the rotor speed x_1 asymptotically converges to $\omega_e^\dagger$ and the rotor acceleration x_2 asymptotically converges to zero, which completes the proof. ■

Corollary 1: The sliding mode controllers u_d, u_q and u_β given in (8), (9) and (14) guarantee that $P_e \to P_e^\dagger$ as $t \to \infty$ provided that $\omega_e > \omega_e^\dagger$. ◇

IV. Simulations

We design sliding mode controllers for a wind turbine with a 2.4MW PMSG. In our case, the rated wind speed is 12m/s and other simulation parameters are given in Appendix. Fig. 2 shows the wind profile. The mean wind speed increases from 11m/s to 14m/s at t=100s and then it keeps over the rated value with the wind turbulence. The turbulence component is assumed to have the Kaimal spectra, and in our simulation the turbulence intensity is 12%. The pitch rate is limited to 8deg/s. The simulation result can be divided into two parts. In the region $\omega_g \le \omega_g^\dagger$, the torque controller tries to maximize the generating power, while in the region $\omega_g > \omega_g^\dagger$, the torque control and pitch control operate in harmony to regulate the inlet power.

In Fig. 3, the performance of proposed controller is verified with the wind profile shown in Fig. 2. Since the rated rotor speed is $\omega_g^\dagger = 142.5$rad/s, it can be said from the figure that before (about) 110s, the torque controller is operating, while after 110s the pitch controller as well as the torque controller is active. Fig. 3c shows that before 110s the power coefficient is almost kept its maximum value $C_{p\,\max} = 0.4382$. Thus, we can say that the torque controller shows good performance against the wind turbulence in the region $\omega_g \le \omega_g^\dagger$.

In the power regulating region (or in the region $\omega_g > \omega_g^\dagger$), the controller should change the pitch angle so that the inlet power is limited to its rated value, which means that ω_g should be regulated to be $\omega_g^\dagger$. The results in Fig. 3 also shows that the generator output power and the mechanical rotor speed are regulated to their rated values after 110s, respectively. It is mainly because of the behavior of the pitch angle in Fig 3d. From the simulation results, it turns out that the proposed controller can successfully regulate the inlet power.

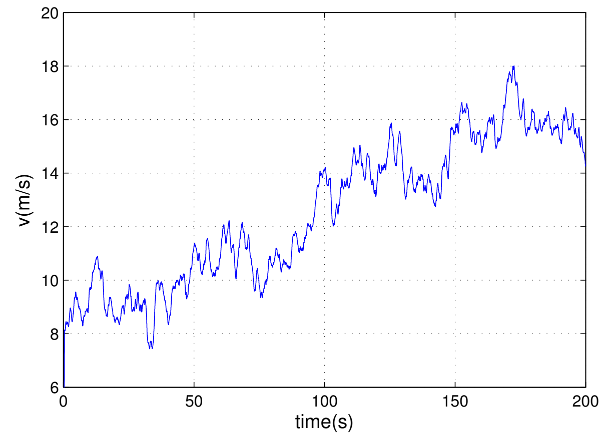

Fig. 2. Wind Speed.

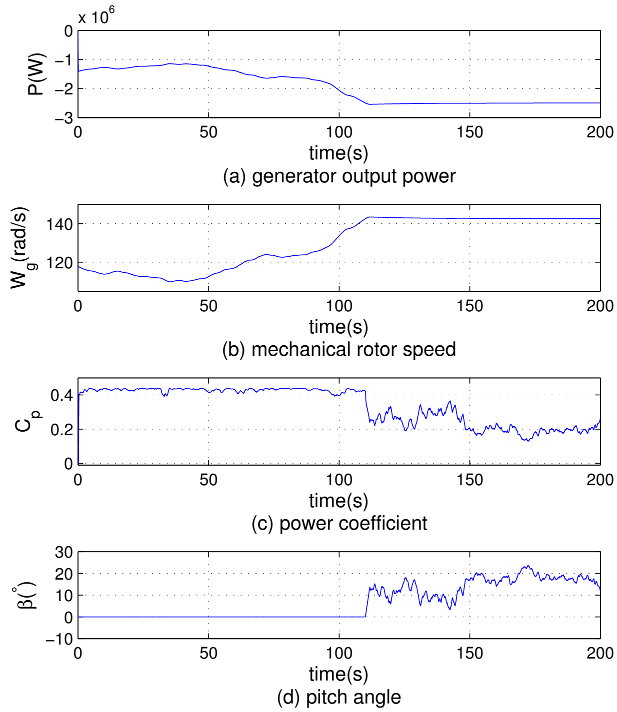

Fig. 3. Simulation Results.

V. Conclusion

In this paper, we proposed a control scheme for the wind turbine system based on a PMSG. An observer is constructed to estimate the turbine torque and the rotor acceleration, and with which a torque controller and a pitch controller are designed employing the sling mode control scheme. In order to verify the proposed control strategy, the stability analysis as well as simulations are performed. Simulation results show that control objectives are well achieved in spite of the wind turbulence. Experimental verification of the proposed scheme is one of important future works.

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APPENDIX

1) Bounds of $C_p(\lambda, \beta)$

We first note that under the condition of $\omega_e \geq \omega_e^\star > 0$ and Assumption 1, it holds that

$$\frac{1}{\Gamma^*} < \frac{1}{\Gamma} < \frac{1}{\Gamma^\dagger}$$

where $\frac{1}{\Gamma^*} = -\frac{0.035}{1+\beta^{\star 3}}$ and $\frac{1}{\Gamma^\dagger} = \frac{NP\overline{v}}{r\omega_e^\star}$.

Moreover, we have

$$\underline{C}_p < C_p < \overline{C}_p$$

where $\underline{C}_p = c_1\left(c_2\frac{1}{\Gamma^*} - 90c_3 - c_4 90^x - c_5\right)e^{-\frac{c_6}{\Gamma^*}}$ and $\overline{C}_p = c_1 c_2 \frac{1}{\Gamma^\dagger} e^{-\frac{c_6}{\Gamma^*}}$. Thus, one has

$$|C_p| \leq C_p^* := \max\{|\underline{C}_p|, |\overline{C}_p|\}.$$

2) Partial derivatives of $C_p(\lambda, \beta)$

$$\begin{aligned}\frac{\partial C_p(\lambda,\beta)}{\partial\lambda} =& \frac{-1}{(\lambda+0.08\beta)^2}e^{-\frac{c_6}{\Gamma}}c_1 \\ &\times\left(c_2 - c_2c_6\frac{1}{\Gamma} + c_3c_6\beta + c_4c_6\beta^x + c_5c_6\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial C_p(\lambda,\beta)}{d\beta} =& e^{-\frac{c_6}{\Gamma}}c_1\Big\{\frac{\partial\frac{1}{\Gamma}}{\partial\beta}\Big(c_2 + c_5c_6 - c_2c_6\frac{1}{\Gamma} \\ &+ c_3c_6\beta + c_4c_6\beta^x\Big) - (c_3 + c_4x\beta^{x-1})\Big\}\end{aligned}$$

where $\frac{\partial\frac{1}{\Gamma}}{\partial\beta} = \frac{-0.08}{(\lambda+0.08\beta)^2} + \frac{0.035\times 3\beta^2}{(1+\beta^3)^2}$.

$$\begin{aligned}\frac{\partial C_p(\lambda(v,\omega_e),\beta)}{\partial\omega_e} =& \frac{-r}{NPv(\lambda+0.08\beta)^2}e^{-\frac{c_6}{\Gamma}}c_1 \\ &\times\left(c_2 - c_2c_6\frac{1}{\Gamma} + c_3c_6\beta + c_4c_6\beta^x + c_5c_6\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial C_p(\lambda(v,\omega_e),\beta)}{\partial v} =& \frac{r\omega_e}{NPv^2(\lambda+0.08\beta)^2}e^{-\frac{c_6}{\Gamma}}c_1 \\ &\times\left(c_2 - c_2c_6\frac{1}{\Gamma} + c_3c_6\beta + c_4c_6\beta^x + c_5c_6\right)\end{aligned}$$

3) System Parameters

Rotor radius $r = 41$ m
Air density $\rho = 1.25\ \mathrm{kg/m^3}$
Resistance $R_s = 0.1\ \Omega$
Inductance $L = 0.005$ H
Field flux $\phi = 10.68\ \mathrm{V\cdot s/rad}$
Equivalent inertia $J = 8000\ \mathrm{kg\cdot m^2}$
Number of pole pairs $P = 2$
Gear ratio $N = 77$
Time constant of pitch actuator $T_\beta = 0.1$ s
Pitch rate limit $R_\beta = 8$ deg/s
C_p curve
$c_1 = 0.22, c_2 = 116, c_3 = 0.4, c_4 = 0$
$c_5 = 5, c_6 = 12.5, x = 0$
$\beta^\star = 0°,\ \lambda_{opt} = 6.325, C_{p\max} = 0.4382$

4) Controller Parameters

Observer
$\mu = 13000, \alpha = 500000, U_M = 20$
Sliding torque controller
$K_d = -1, K_q = -10, \gamma_1 = -1$
Sliding pitch controller
$\gamma_2 = 0.1, \hat{v} = 15$ m/s, $K_\beta = -5000$
$\omega_g^\dagger = 142.5$ rad/s

REFERENCES

[1] M. R. Patel, *Wind and Solar Power Systems Design, Analysis, and Operation*, New York: Taylor & Francis, chap. 2, 2006.
[2] W. E. Leithead and B. Connor, "Control of variable speed wind turbines: design task," *Int. Journal of Control*, Vol. 73, No. 13, pp. 1189-1212, Sep. 2000.
[3] T. Senjyu, N. Nakasone, A. Yona, A.Y. Saber, T. Funabashi, and H. Sekine, "Operation strategies for Stability of gearless wind power generation systems," *in Proc. IEEE Power and Energy Society General Meeting 2008*, Jul. 20-24, 2008.
[4] M. Yin, G. Li, M. Zhou, and C. Zhao, "Modeling of the wind turbine with a permanent magnet synchronous generator for integration," *in Proc. IEEE Power and Energy Society General Meeting 2007*, Jun. 24-28, 2007.
[5] F. Valenciaga, P. F. Puleston, P. E. Battaiotto, and R. J. Mantz, "Passivity/sliding mode control of a stand-alone hybrid generation system," *IEE Proceedings - Control Theory and Applications*, Vol. 147, No. 6, pp. 680-686, Nov. 2000.
[6] B. Beltran, T. Ahmed-Ali, and M. E. H. Benbouzid, "Sliding mode power control of variable-speed wind energy conversion systems," *IEEE Transactions on Energy Conversion*, Vol. 23, No. 2, pp. 551-558, Jun. 2008.
[7] F. Valenciaga and P. F. Puleston, "High-order sliding control for a wind energy conversion system based on a permanent magnet synchronous generator," *IEEE Transactions on Energy Conversion*, Vol. 23, No. 3, pp. 860-867, Sep. 2008.
[8] B. Beltran, T. Ahmed-Ali, and M. E. H. Benbouzid, "High-order sliding-mode control of variable-speed wind turbines," *IEEE Transactions on Industrial electronics*, Vol. 56, No. 9, pp. 3314-3321, Sep. 2009.
[9] B. Wang and S. Qin, "Backstepping sliding mode control of variable pitch wind power system," *in Proc. 2010 Asia-Pacific Power and Energy Engineering Conference*, Mar. 28-31, 2010.
[10] C. Zhu, R. Zhou, and Y. Wang, "A new nonlinear voltage controller for power systems," *Electrical Power and Energy Systems*, Vol. 19, No. 1, pp. 19-27, Jan. 1997.
[11] Z. Lubosny, *Wind Turbine Operation in Electric Power Systems*, Berlin: Springer, chap. 5, 2003.
[12] A. Levant, *Introduction to high-order sliding modes*, http://www.tau.ac.il/~levant/hosm2002.pdf, 2002.

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