

# Fuzzy PD Speed Controller for Permanent Magnet Synchronous Motors

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## Abstract

This paper presents a fuzzy PD speed control scheme for the robust speed tracking of a permanent magnet synchronous motor (PMSM). Motivated by the common control engineering knowledge that transient performance can be improved if the P gain is big and the D gain is small in the beginning, a linearizing control scheme with a fuzzy PD controller is proposed. The global system stability is analyzed and the proposed control algorithm is implemented using a TMS320F28335 DSP. Simulation and experimental results are given to verify the effectiveness of the proposed method.

**Key Words:** Fuzzy control, PD control, Permanent magnet synchronous motor (PMSM), Robust control, Speed control

## I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are extensively used in high-performance drives such as servo and traction applications owing to the advantages of a high power density, a high efficiency, low inertia, and low noise [1]-[8]. However, PM synchronous motors are not easily controlled due to their nonlinear characteristics and uncertainties such as system parameter variations, unmodeled dynamics and external load disturbances. To solve these problems several researchers such as [4]-[8] have recently proposed fuzzy control design methods for a permanent magnet synchronous motors. Fuzzy control theory provides an alternative tool for collecting human knowledge and for dealing with nonlinearities or uncertainties, and it has succeeded in controlling complex nonlinear or uncertain systems that are not amenable to conventional control methods [9], [10]. The PMSM fuzzy control design methods presented in [4]-[8], and the references therein are based on the heuristics-based fuzzy approach which is essentially model free. However, as pointed out in [9] these methods can have two major drawbacks. They do not have a systematic and tractable design technique, and thus analysis of stability, performance or robustness of the control system is very difficult.

Taking these facts into account a nonlinear control scheme for the robust speed control of a PM synchronous motor is proposed. The proposed speed control law consists of two controllers: a linearizing controller and a fuzzy PD

controller. The linearizing controller linearizes the nonlinear PMSM model, while the fuzzy PD controller stabilizes the linearized dynamics. The PD parameters are chosen based on the common control engineering knowledge that transient performance can be improved if the  $P$  gain is big and the  $D$  gain is small in the beginning [10]. Unlike the previous fuzzy control design methods of [4]-[8], the global stability of the proposed control system can be guaranteed. By using the results from [11], the asymptotic stability is proven. Simulation and experimental results are given to confirm the feasibility of the proposed control method.

## II. PMSM MODEL DESCRIPTION

In the rotor flux oriented synchronous rotating reference frame, a surface-mounted permanent magnet synchronous motor (PMSM) can be represented by the following equation:

$$\begin{aligned}\dot{\omega} &= k_1 i_{qs} - k_2 \omega - k_3 T_L \\ \dot{i}_{qs} &= -k_4 i_{qs} - k_5 \omega + k_6 V_{qs} - \omega i_{ds} \\ \dot{i}_{ds} &= -k_4 i_{ds} + k_6 V_{ds} + \omega i_{qs}\end{aligned}\quad (1)$$

where,  $T_L$  represents the load torque,  $\omega$  is the electrical rotor angular speed,  $i_{qs}$  is the q-axis current,  $V_{qs}$  is the q-axis voltage,  $i_{ds}$  is the d-axis current,  $V_{ds}$  is the d-axis voltage, and  $k_i > 0$ ,  $i = 1, \dots, 6$  are the parameter values depending on the stator resistance  $R_s$ , the stator inductance  $L_s$ , the number of poles  $p$ , the equivalent rotor inertia  $J$ , the viscous friction coefficient  $B$ , and the magnetic flux  $\lambda_m$ .

$$\begin{aligned}k_1 &= \frac{3}{2} \frac{1}{J} \frac{p^2}{4} \lambda_m, \quad k_2 = \frac{B}{J}, \quad k_3 = \frac{p}{2J}, \\ k_4 &= \frac{R_s}{L_s}, \quad k_5 = \frac{\lambda_m}{L_s}, \quad k_6 = \frac{1}{L_s}\end{aligned}$$

In this paper, the following assumptions are used to design the fuzzy PD speed controller:

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A1:  $\theta$ ,  $\omega$ ,  $i_{qs}$ , and  $i_{ds}$  are available.

A2:  $\dot{T}_L$  can be neglected and it can be set to  $\dot{T}_L = 0$ .

A3: The desired speed  $\omega_d$  is twice differentiable and  $\omega_d$ ,  $\dot{\omega}_d$ , and  $\ddot{\omega}_d$  are bounded.

It should be noticed that almost all of the previous control law design methods were based on the above assumptions.

### III. FUZZY PD SPEED CONTROLLER DESIGN AND STABILITY ANALYSIS

#### A. Linearized Error Dynamics

The control inputs  $V_{qs}$  and  $V_{ds}$  can be decomposed as:

$$\begin{aligned} V_{qs} &= \frac{1}{k_1 \cdot k_6} (u_{fbq} + u_{fq}) \\ V_{ds} &= \frac{1}{k_6} (u_{fbd} + u_{fd}) \end{aligned} \quad (2)$$

Based on the feedback linearization method given in [12] and [13], the linearizing control terms  $u_{fq}$  and  $u_{fd}$  can be computed by:

$$\begin{aligned} u_{fq} &= \ddot{\omega}_d + k_2\alpha + k_1k_4i_{qs} + k_1k_5\omega + k_1\omega i_{ds} \\ u_{fd} &= k_4i_{ds} - \omega i_{qs} \end{aligned} \quad (3)$$

where,  $\alpha = k_1i_{qs} - k_2\omega - k_3T_L = \dot{\omega}$ . The linearizing control law results in the following linear time-invariant error dynamics:

$$\dot{\omega}_e = \alpha_e, \quad \dot{\alpha}_e = u_{fbq}, \quad \dot{i}_{ds} = u_{fbd}$$

where,  $\omega_e = \omega - \omega_d$  and  $\alpha_e = \dot{\omega}_e = \alpha - \dot{\omega}_d$ . If  $u_{fbq}$  and  $u_{fbd}$  are set to:

$$u_{fbq} = -K_P\omega_e - K_D\frac{d}{dt}\omega_e, \quad u_{fbd} = -K_3i_{ds} \quad (4)$$

Then the closed-loop control system is given by:

$$\dot{e} = \begin{bmatrix} 0 & 1 & 0 \\ -K_P & -K_D & 0 \\ 0 & 0 & -K_3 \end{bmatrix} e \quad (5)$$

where,  $e = [\omega_e, \alpha_e, i_{ds}]^T$ , and the characteristic function of the closed-loop control system is given by the following third-order polynomial:

$$s^3 + a_2s^2 + a_1s + a_0 \quad (6)$$

where,  $a_2 = K_D + K_3$ ,  $a_1 = K_3K_D + K_P$ , and  $a_0 = K_3K_P$ . In many previous papers such as [6], [14], [15], the dynamic characteristics of  $i_{ds}$  are neglected to simplify the stability analysis under the assumption that the  $d$ -axis current controller works well and that the output  $i_{ds}$  satisfies  $i_{ds}=0$ . However, in this paper, the dynamics of  $i_{ds}$  are considered for a rigorous stability analysis, as well as the PD control term  $u_{fbq}$  and the linearizing control term  $u_{fq}$ .

#### B. Fuzzy PD Speed Controller

The common rule given in [10] implies that the transient performance can be improved if the  $P$  gain is big and the  $D$  gain is small when the error is big. Considering this rule the conventional PD controller (4) was modified. The proposed fuzzy PD control input  $u_{fb}$  is determined by five fuzzy rules of the following form:

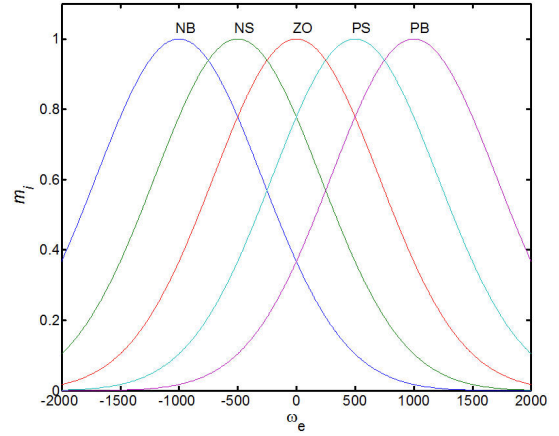


Fig. 1. Membership functions.

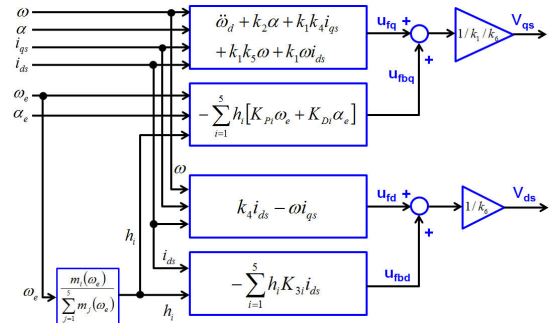


Fig. 2. Block diagram of the proposed fuzzy PD speed controller.

**Rule  $i$ :** IF  $\omega_e$  is  $F_i$ , THEN  $u_{fbq} = -K_{P_i}\omega_e - K_{D_i}\alpha_e$ ,  $u_{fbd} = -K_{3_i}i_{ds}$

where,  $F_i$  ( $i=1, \dots, 5$ ) are the fuzzy sets,  $K_{P_1} \geq K_{P_2} \geq K_{P_3} > 0$ ,  $K_{D_3} \geq K_{D_2} \geq K_{D_1} > 0$ ,  $K_{3_1} \geq K_{3_2} \geq K_{3_3} > 0$ ,  $K_{P_5} \geq K_{P_4} \geq K_{P_3} > 0$ ,  $K_{D_3} \geq K_{D_4} \geq K_{D_5} > 0$ , and  $K_{3_5} \geq K_{3_4} \geq K_{3_3} > 0$ . The membership functions  $m_i$  according to the speed error  $\omega_e$  is shown in Fig. 1. In this figure, ZO ( $F_3$ ) stands for zero, PS ( $F_4$ ) for positive small, PB ( $F_5$ ) for positive big, NS ( $F_2$ ) for negative small, and NB ( $F_1$ ) for negative big.

By using a standard fuzzy inference method (using a singleton fuzzifier, a product fuzzy inference and a weighted average defuzzifier), the final fuzzy PD control input is obtained as:

$$\begin{aligned} u_{fbq} &= -\sum_{i=1}^5 h_i \left[ K_{P_i}\omega_e + K_{D_i}\frac{d}{dt}\omega_e \right], \\ u_{fbd} &= -\sum_{i=1}^5 h_i K_{3_i}i_{ds} \end{aligned} \quad (7)$$

where,  $h_i = m_i / \sum_{j=1}^5 m_j$  is the normalized weight of each IF-THEN rule and it satisfies  $h_i \geq 0$  and  $\sum h_i = 1$ . Fig. 2 shows a block diagram of the proposed fuzzy PD speed controller.

#### C. Stability Analysis

Using (1), (3), and (7) the following error dynamics can be obtained:

$$\dot{e} = \sum_{i=1}^5 h_i \begin{bmatrix} 0 & 1 & 0 \\ -K_{P_i} & -K_{D_i} & 0 \\ 0 & 0 & -K_{3_i} \end{bmatrix} e \quad (8)$$

TABLE I  
 PMSM SPECIFICATIONS

Parameters	Items
Number of poles ( $p$ )	12
Stator resistance ( $R_s$ )	0.99 [ $\Omega$ ]
Stator inductance ( $L_s$ )	5.82 [ $mH$ ]
Magnetic flux ( $\lambda_m$ )	0.079153 [ $V \cdot sec/rad$ ]
Equivalent inertia ( $J$ )	0.00120754 [ $kg \cdot m^2$ ]
Viscous friction coefficient ( $B$ )	0.0003 [ $N \cdot m \cdot sec/rad$ ]
Load torque ( $T_L$ )	0.7 [ $N \cdot m$ ]

which yields the characteristic function (6) with:

$$\begin{aligned} a_2 &\in [K_{D0} + K_{33}, K_{D3} + K_{30}], \\ a_1 &\in [K_{33}K_{D0} + K_{P3}, K_{30}K_{D3} + K_{P0}], \\ a_0 &\in [K_{33}K_P, K_{30}K_{P0}] \end{aligned} \quad (9)$$

where,  $K_{P0} = \max\{K_{P1}, K_{P5}\}$ ,  $K_{D0} = \min\{K_{D1}, K_{D5}\}$ , and  $K_{30} = \max\{K_{31}, K_{35}\}$ . By [11] it can be seen that the characteristic function (6) with (9) is asymptotically stable if:

$$(K_{D0} + K_{33})(K_{33}K_{D0} + K_{P3}) > K_{P0}K_{30} \quad (10)$$

This proves the following theorem:

**Main Theorem:** Consider the closed-loop system of (1) and (2) with the linearizing control law (3) and the fuzzy PD control law (7). Then, the asymptotic stability of  $e = 0$  is guaranteed as long as the PD parameters satisfy the conditions of (10).

#### IV. SIMULATION AND EXPERIMENTAL RESULTS

For the simulations and experiments, a PMSM (1) with the system parameters given in Table I was considered. Thus, the PMSM model equations can be rewritten as follows:

$$\begin{aligned} \dot{\omega} &= 3539.6i_{qs} - 0.2484\omega - 4968.8T_L \\ \dot{i}_{qs} &= -170.1i_{qs} - 13.6\omega + 171.8V_{qs} - \omega i_{ds} \\ \dot{i}_{ds} &= -170.1i_{ds} + 171.8V_{ds} + \omega i_{qs} \end{aligned} \quad (11)$$

Using (3), the linearizing control terms  $u_{fq}$  and  $u_{fd}$  are given by:

$$\begin{aligned} u_{fq} &= \ddot{\omega}_d + 0.2484\alpha + 3539.6(170.1i_{qs} + 13.6\omega + \omega i_{ds}) \\ u_{fd} &= 170.1i_{ds} - \omega i_{qs} \end{aligned} \quad (12)$$

To obtain the fuzzy PD control law (7) that stabilizes the linearized error dynamics, the membership function and PD gains which satisfy the stability condition (10) are:

$$\begin{aligned} m_i &= e^{-\mu(\omega_e - W_i)^2}, \\ W_1 &= -W_5 = -1000, W_2 = -W_4 = 0.5W_1, W_3 = 0, \\ \mu &= 1 = W_1^2, \\ K_{P1} &= K_{P5} = 70000, K_{P2} = K_{P4} = 65000, K_{P3} = 50000, \\ K_{D1} &= K_{D5} = 100, K_{D2} = K_{D4} = 400, K_{D3} = 600, \\ K_{31} &= K_{35} = 700, K_{32} = K_{34} = 600, K_{33} = 500. \end{aligned} \quad (13)$$

For a comparative study, a linearizing control scheme with the conventional non-fuzzy PD controller (4) was also carried out. In this case, the PD gains are given by:

$$K_P = K_{P1} = 70000, K_D = K_{D1} = 100, K_3 = K_{31} = 700. \quad (14)$$

Fig. 3 shows the overall control system which consists of a PMSM, an encoder, a brake for the load torque, and a three-phase PWM inverter with a TMS320F28335 DSP controller.

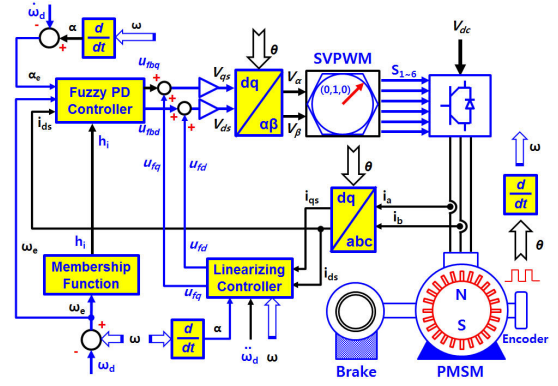


Fig. 3. Overall block diagram of the proposed control system.

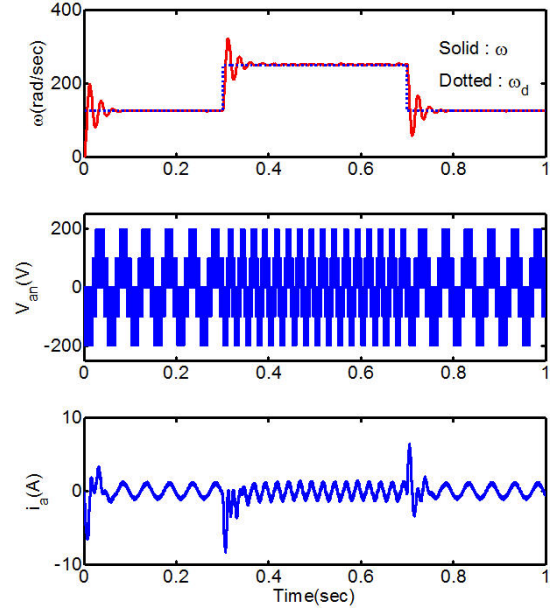


Fig. 4. Simulation results with the non-fuzzy PD controller.

In this paper, the switching frequency is chosen as 5 [kHz], and a space vector PWM (SVPWM) technique is used to realize the control input applied to the motor. To evaluate the control performance under a step speed change, the reference speed ( $\omega_d$ ) increases from 125.66 [rad/sec] to 251.33 [rad/sec] and then decreases from 251.33 [rad/sec] to 125.66 [rad/sec].

The proposed control algorithm is simulated using Matlab/Simulink for two cases: a conventional non-fuzzy PD controller (14) and the proposed fuzzy PD controller (13). Figs. 4 and 5 show the simulation results for the speed response with a conventional non-fuzzy PD controller and the proposed fuzzy PD controller, respectively. As can be seen in the figures, the proposed PD method assures a better transient performance such as no overshoot and a fast settling time when compared with the conventional PD method. Fig. 6 shows the experimental test bed, which includes a three-phase inverter with a TI DSP TMS320F28335, a surface-mounted PMSM, and a brake for the load torque. Figs. 7 and 8 show the experimental results under the same condition as Figs. 4 and 5, respectively. Figs. 7 (a) and 8 (a) show the desired speed ( $\omega_d$ ) and the measured speed ( $\omega$ ) under a conventional non-fuzzy PD controller and the proposed fuzzy PD controller, respectively. Figs. 7 (b) and 8 (b) show the line to neutral

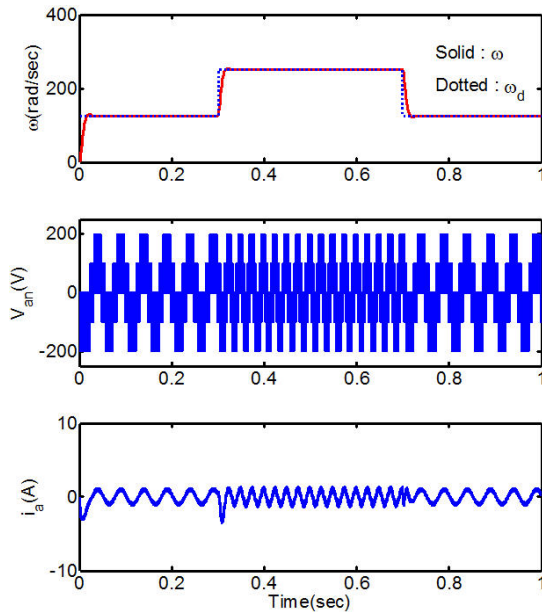


Fig. 5. Simulation results with the proposed fuzzy PD controller.

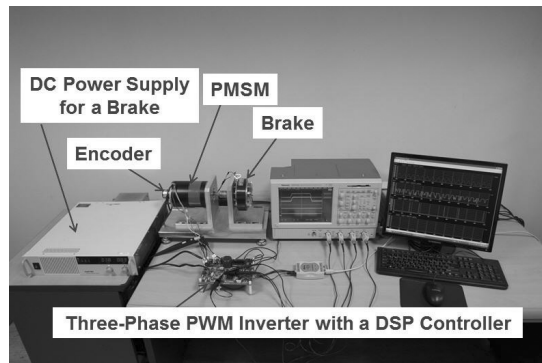


Fig. 6. Experimental test bed.

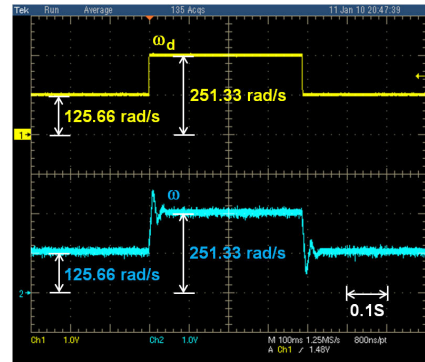
voltage ( $V_{an}$ ) and the phase  $a$  current ( $i_a$ ) under a conventional non-fuzzy PD controller and the proposed fuzzy PD controller, respectively. From the simulation and experimental results, it can be seen that the control performance of the proposed fuzzy PD control law is much better than that of the conventional non-fuzzy PD controller.

## V. CONCLUSIONS

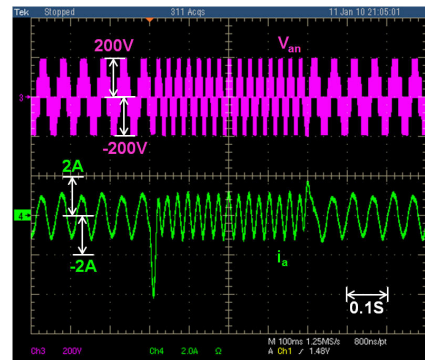
Motivated by the common control engineering knowledge that transient performance can be improved if the control input is big at the beginning, a linearizing control scheme with a fuzzy PD controller was proposed. Also, the stability condition, which guarantees the asymptotic stability of the overall system, was analytically derived. Finally, the effectiveness of the proposed method was verified by simulation and experimental results.

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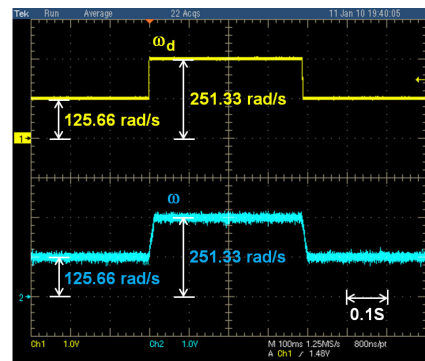


(a) Desired speed ( $\omega_d$ ) and measured speed ( $\omega$ ).

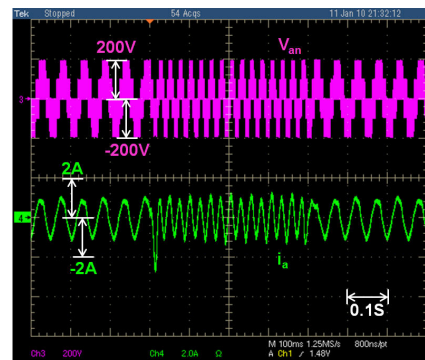


(b) Line to neutral voltage ( $V_{an}$ ) and phase  $a$  current ( $i_a$ ).

Fig. 7. Experimental results with the non-fuzzy PD controller.



(a) Desired speed ( $\omega_d$ ) and measured speed ( $\omega$ ).



(b) Line to neutral voltage ( $V_{an}$ ) and phase  $a$  current ( $i_a$ ).

Fig. 8. Experimental results with the proposed fuzzy PD controller.



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